

# Epstein Explains Einstein

An Introduction to both the Special and the General Theory of Relativity

by David Eckstein



“As simple as possible - but not simpler !”



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Translation of the German edition by Samuel Edelstein

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## Foreword by Lewis C. Epstein

Who contributes the most to progress: The mountain man who finds the first pass at 10,000 feet through a high range, or the railroad engineer who later finds a low pass at 7,000 feet which will be used by trains, motor vehicles, pipelines, electric lines and optical cables?

It is hard to answer.

In the realm of physics, all the credit, the Nobel Prize, goes to the "mountain man". In physics, the "railroad engineer's" reward is best some money from writing and selling "low pass maps", which are books that make good understanding accessible to those who for various reasons cannot go up to 10,000 feet.

I am a "railroad engineer". I found a low pass through the theoretical physics mountains into Einstein land. The pass is mapped in a picture and a story book called *Relativity Visualized*. Here David Eckstein has taken my picture story and transliterated it into kosher physics. The story pivots on an intuitive idea I called: the speed of time.

From where came this "speed of time" story? Like many post 1960 physics ideas, it just came out of the smoke, which opens the band pass filter of the mind. Through the open filter comes lots of noise, a few distant memories and unusual convolutions of thought.

Recall childhood. I remember how slow time crawled when I was kept in detention, after school going home time, because of bad spelling or bad goofing. And I remember how fast time flew on the special occasion of riding up front on a steam locomotive's footplate. The Ancients too felt earthly time ran slow and fast. Slow in summer; each daylight hour became longer. Fast in winter; each daylight hour become shorter. Even in the lower spheres of heaven, the planets pace through the zodiac was not only variable, but occasionally back stepping.

Galileo put his first thermometer into chile pepper and demonstrated that part of what had been called heat was subjective, not objective. And he suspected time might also be, in part, subjective. So he tried hard to express the falling body law in terms of objective geometry, on distance from the top. Only reluctantly did he permit time to enter the falling body law. After all, how could time, a thing without material existence, have a linear control of a material object's speed? Time was not part of the tangible world. The Good Book relates how God created the world: 1) In the beginning God created the heaven and the earth, 2), 3), 4), 5), 6), 7) And on the seventh day he took a rest. On which day did God create time? Answer: The Ancients did not think time had objective existence, so it need not to be created. Time came out of men's head.

But once permitted into physics, time soon established itself as the immutable universal independent variable which drives all physical processes. The current of time, unalterable, untouchable by any force, any motion, any environment, anything whatsoever, ruled the dynamic world for the three centuries after Galileo.

No sooner had this immaculate conception of time set hard in human intuition than along came Einstein's wild idea: different, equally valid times can simultaneously coexist in the same space. The universal independent variable view of time was only three centuries old when Einstein arrived. Three centuries is brief when you realize the ancient view of time had sufficed for six hundred centuries.

If different times can coexist, then something like the child's view of time, something akin to the ancient view, is reopened. Different times can run at different speeds relative to each other. And so the words "speed of time" are reinflated with life. What follows in this work is David Eckstein's perspective on the new life and its immediate consequences.

San Francisco, California, Summer of 2008

Lewis Carrol Epstein

## Foreword by the Author

I trust to have found a way of introducing the special *and* the general theory of relativity to high-school and undergraduate students, treating most of the aspects quantitatively. A certain familiarity with elementary physics is required. Occasionally calculus at high school level comes into this presentation. However, you can delegate these calculations to an modern pocket calculator - or just put your faith in the results presented by the author.

In any case this book is not primarily about calculations. On the contrary, the aim is to develop a profound 'Anschauung', an insight into the concepts of Einstein's theories. To this end the diagrams of Lewis C. Epstein are used to visualize the phenomena. My experience is, that, after a short time, students think of these Epstein diagrams as self-evident ! Epstein diagrams are quantitatively correct, and they are simpler to draw and to read than the widely used diagrams of the Minkowski type. Furthermore, they also help us to visualize fundamental aspects of the general theory of relativity. Beginning with the basic phenomena (the relativity of synchronicity, the relativity of duration and the relativity of length), we proceed carefully, step by step, to the more abstract Lorentz transformations, eventually arriving at the Schwarzschild metric. We will be able to verify most of the famous experiments performed to test the special and the general theory of relativity.

Each chapter concludes with a page of "problems and suggestions". The detailed solutions to these problems would require more than a hundred pages in print, thus creating a monstrous and expensive book. However, these solutions (as well as other material connected with the topic) are available to everyone at "[www.relativity.li](http://www.relativity.li)". Levin Gubler has designed this beautiful website for me in such a way I needed only provide the contents. I would like to take the opportunity now to cordially thank him for his work.

This book would not exist were it not for the assistance of many individuals. First of all I think of my high-school students whose reactions forced me to reconsider certain aspects and reformulate them. Then, in particular, I am indebted to the authors of two books: One of them is the well-known Lewis C. Epstein, the other is Horst Melcher, whose name is hardly familiar to our readers. Together they helped *me* to achieve a deeper understanding of the subject.

I also owe a dept of gratitude to three friends of mine, Alfred Hepp, Hans Buchmann and Hans Walser. They did a critical reading of early versions of the text, and their input contributed vastly to an improvement in style and content. Alfred Hepp and Jonathan Gubler helped me to improve the visual layout of the book, making it more attractive to the eye.

This English version of the book would never have been born without the many hours Samuel Edelstein invested to translate it from German into English. I would like to thank him cordially for his work! However, I take full responsibility for any errors and 'strange' formulations that readers may encounter.

Further I would like to thank the Canton Thurgau and its tax payers. This small state of the Federal Republic of Switzerland provided me through means of a sabbatical the free time necessary to convert my accumulated documents and experiences into this book. It would be nice to think that many colleagues worldwide as well as their students should find it useful.

I would like to thank the ESO and the CERN for their friendly permission to use copyright protected pictures. The same goes for the cartoonists Sidney Harris and Oswald Sigg. "Insight-Press" in San Francisco generously granted permission to print the many drawings from Epstein's book. The former "Hamburgische Electricitäts-Werke" (now Vattenfall Europe AG) granted permission for the use of the illustrations in section **F3**. Also Franz Embacher in Vienna has kindly allowed me to use his illustration of the Thirring-Lense-Effect.

I do admit that I have downloaded some pictures from the internet without legal clarification. Other illustrations were provided to me by students without indication of source (e.g. the autostereogram at the end of **E6**). For most illustrations the source is however indicated. Of course many of the drawings, photographs and computer graphics come from the author.

Frauenfeld, mid of march 2007	(German version)	
Frauenfeld, end of september 2009	(English version)	"David Eckstein"

Many careful readers of the previous versions of my book have contributed a lot of corrections and improvements to the actual third version. Thanks to them all !

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Spiral Galaxy NGC 4565  
(FORS / VLT)

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## A Storm Clouds Gather

The 'perennial foundations of thinking' are presented: Newton's concepts of space, time and inertial mass. Also following Newton we define inertial frames of references. Next we present Galileo's formulation of the relativity of uniform movement. The principle of relativity is formulated based on the concept of an inertial frame. In the third section we show that Maxwell's theory of electromagnetism does not fit with Newton's mechanics and Galileo's equivalence of all inertial frames: Newton and Galileo's manner of adding velocities shows itself in conflict with the constancy of the speed of light. The fourth section presents Einstein's approach to solving this conflict. Finally we take a short detour to other fields of activity, for which around 1900 storm clouds began to gather.

## A1 Newtonian Foundations of Classical Physics

Newton published his great work “Philosophiae naturalis principia mathematica” in 1687 ([01], English edition [02] or [03]). In this work he used Euclidean geometry to embed the previous work of the ‘giants’ Kepler, Galileo, Descartes and Huygens into a uniform and a comprehensive theory, which today we call Newtonian mechanics. Kepler’s laws of planetary motion, Galileo’s law of relativity of all uniform motion, Descartes’ law of conservation of momentum, Huygen’s analysis of circular motion, and the motion of the heavenly and terrestrial bodies were all reduced to 3 axioms and 1 force law. In his book Newton successfully applies his theory to compute the flattening of the earth and of Jupiter, to justify the tides and much more.

The success of his mathematical ideas affected the course of human thought far beyond the natural sciences, and the following two centuries only increased this success. It was therefore even more difficult to abandon the fundamental conceptions on which Newton’s theory builds – the concepts of time, of space and of the inertial mass of a body.

### Newtonian Time

The master formulated it beautifully in his book [03-408]:

“Although time, space, place and motion are very familiar to everyone, it must be noted that these quantities are popularly conceived solely with reference to the objects of sense perception. And this is the source of certain preconceptions; to eliminate them it is useful to distinguish these quantities into absolute and relative, true and apparent, mathematical and common.

Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. Relative, apparent, and common time is any sensible and external measure (precise or imprecise) of duration by means of motion; such a measure – for example, an hour, a day, a month, a year – is commonly used instead of true time.”

And later [03-410]:

“In astronomy, absolute time is distinguished from relative time by the equation of common time. For natural days, which are commonly considered equal for the purpose of measuring time, are actually unequal. Astronomers correct this inequality in order to measure celestial motions on the basis of a truer time. It is possible that there is no uniform motion by which time may have an exact measure. All motions can be accelerated and retarded, but the flow of absolute time cannot be changed.”

Newton’s concept of a true, absolute, mathematical time applies for all observers at all places equivalently. Time flows continuously and regularly. Its course cannot be affected by heat, acceleration or gravity. Two different observers always measure the same time interval for the same procedure (up to inaccuracies due to the incompleteness of their clocks). Atomic clocks tick in the laboratory exactly as they would in orbit. And a clock is perfect, if the first derivative of the indicated time to true time is constant, that is when the second derivative is zero.

This true and absolute time implies that the simultaneity of two events is an absolute fact. It is independent of the location or of the state of movement of the observers. Time flows the same for all, just as the sun shines the same on all, just and unjust alike.

## Newtonian Space

Concerning space and location things are somewhat more complicated, although Newton likewise postulates a true and absolute space [03-408f]:

“Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space, as in the case of space under the earth or in the air or in the heavens, where the dimension is determined from the situation of the space with respect to the earth. Absolute and relative space are the same in species and in magnitude, but they do not always remain the same numerically. For example, if the earth moves, the space of our air, which in a relative sense and with respect to the earth always remains the same, will now be one part of the absolute space into which the air passes, now another part of it, and thus will be changing continually in an absolute sense.”

And somewhat later [03-410]:

“Just as the order of the parts of time is unchangeable, so, too, is the order of the parts of space. Let the parts of space move from their places, and they will move (so to speak) from themselves. For times and spaces are, as it were, the places of themselves and of all things. All things are placed in time with reference to order of succession and in space with reference to order of position. It is of the essence of spaces to be places, and for primary places to move is absurd. They are therefore absolute places, and it is only changes of position from these places that are absolute motions.”

Contrary to absolute time, to which we are helplessly subjugated, we can freely move in absolute space. Newton clearly sees that one must consider three cases:

1. Accelerated movement along a straight line: This is easily identified by the force of inertia it gives rise to.
2. Rotation relative to absolute space: This is recognizable by centrifugal force, to which it gives rise. It is this absoluteness of the rotation (rotation relative to what exactly??), which convinced Newton of the existence of absolute space. In his famous description of the bucket experiment [03-412f] he stresses that he himself performed this experiment.
3. Uniform movement along a straight line: This is characterized in that no additional forces arise. Therefore, in principle, it cannot be determined whether one is at rest in absolute space or whether one is moving with constant speed.

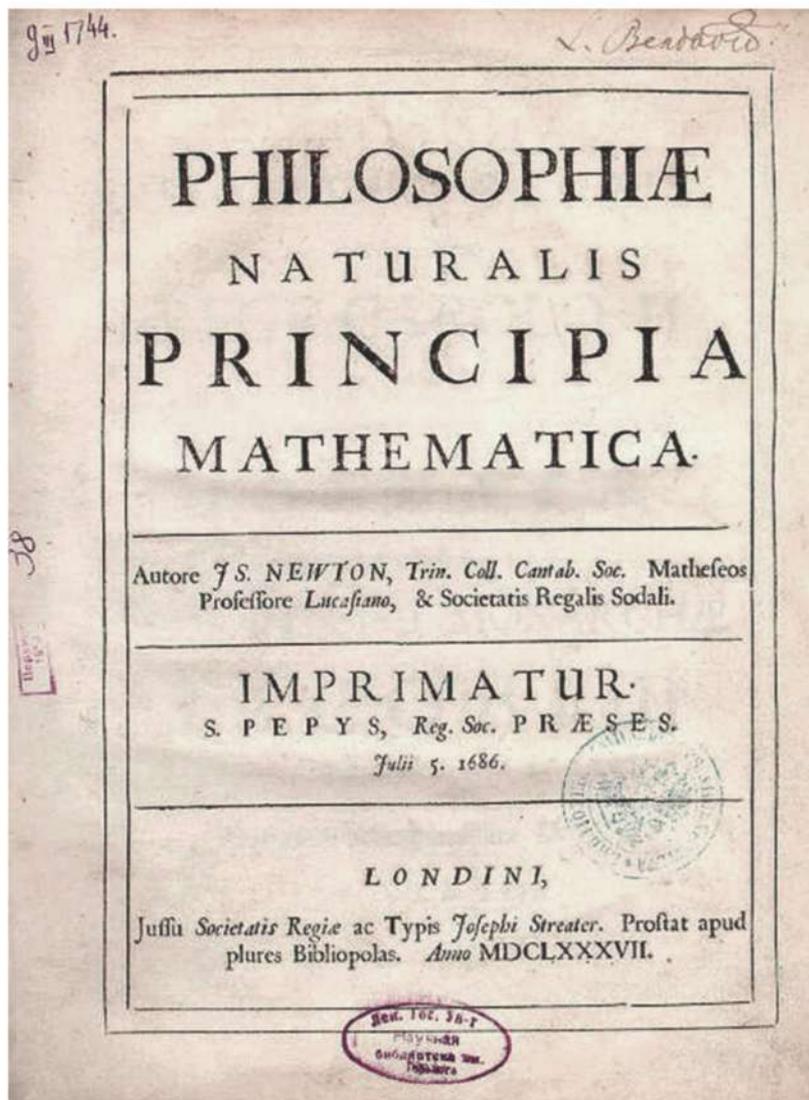
This leads us to the important idea of the *inertial frame*.

## Inertial Frame of Reference

Spatial coordinate systems, which are not accelerated and which do not rotate, are called inertial frames of reference (or simply inertial frames). These are the coordinate systems, which rest in Newton's absolute space or which move uniformly therein. Such coordinate systems are suitable for describing mechanical processes without the need to introduce 'fictitious forces' (also called 'pseudo forces' or 'inertial forces').

An often found definition of an inertial frame is the following: Inertial frames are coordinate systems, which do not move relative to the fixed stars. Why is this definition, if taken as stated, useless?

Lengths, distances and angles can be measured however in arbitrary (i.e., non-inertial) coordinate systems. All observers measure the same value, just as they would measure the same length of time for a dynamic event (e.g., a roof tile falling to the road). The length of an object depends in no way on how fast it is moving.



## Newtonian Mass

Each material body possesses a quantity of mass, with which the body resists to being accelerated:  $\mathbf{F} = m \cdot \mathbf{a}$ . The force needed for a given acceleration  $\mathbf{a}$  is a direct measure of this inertial mass  $m$  of the body. Newton differentiates carefully between inertial mass and gravitational mass, and he performed his own experiments, to verify that the inertial mass and the weight of a body are always proportional to each other:

“... I mean this quantity whenever I use the term ‘body’ or ‘mass’ in the following pages. It can always be known from a body’s weight. For - by making very accurate experiments with pendulums - I have found it to be proportional to the weight, as will be shown below.” [03-404]

This material quantity is, of course, independent of the movement of the body, just like it is independent of the air pressure or of the temperature of the body. The mass of a body is a constant, which is assigned to it, as long as it is not divided in any way.

The fact that the inertial and the gravitational mass of a body are proportional to each other (whereby the constant of proportionality depends only on the selected units and therefore could also be 1) was a fact Newton could not explain and which he even distrustfully questioned with his experiments [03-700ff]. In section **G2** we will see that for Einstein the equality of the inertial and gravitational mass is no longer a fact to be explained but, rather, a fundamental axiom of a new theory.

Space, time and mass are the fundamental ideas, on which Newton developed his mechanics. All other physical quantities can be derived from these three (e.g. try it yourself for the pressure!). The fact that space, time and mass are fundamental is reflected also in the fact that the associated units (second, meter and kilogram) were until 1983 base quantities. This also explains why pendulum, yardstick and weight stone are the Insignia of the American Institute of Physics.



In the context of relativity theory all three of these basic quantities will actually turn out to be ‘relative’; little remains of Newton’s absolute time and absolute space. Also the independence of the inertial mass from the reference system must be abandoned. A somewhat hard formulation would be: the basic assumptions of Newton turned out to be prejudices. To recognize this after the enormous success of Newtonian mechanics required a certain boldness!

## A2 Galileo Galilei's Principle of Relativity

Newton was not the first to state that it is impossible to decide whether an object or a coordinate system in absolute space is moving or at rest. We all know the situation from everyday life: Is it our train or is it the one on the tracks next to us that is moving?

Galileo Galilei described this fact, in his typically colorful language, in his famous "Dialogue Concerning the Two Chief World Systems" and was probably not even the first. The original Italian (!) edition [04] appeared in 1632 and was translated in 1641 into Latin. A German edition, anno 2006, is out of print. English, however, offers several different editions. We follow the translation of Drake:

"Shut yourself up with some friend in the main cabin below decks on some large ship and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide [Note David Eckstein/Samuel Edelman: This should rather read 'narrow' instead of 'wide'. Think of a bottle with a narrow neck on its top! ] vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. ..." [05-187f]

We can summarize this somewhat more soberly:

If a coordinate system B moves uniformly (with constant velocity) in a straight-line with respect to an inertial frame A, then B is also an inertial frame. Or: Two inertial frames can move only uniformly along a straight-line to each other. It cannot be recognized whether one of the two is at rest in absolute space.



### A3 Incompatibility of Maxwell's Theory of Electromagnetism

Einstein was always thorough in his thought and argumentation. In this section we investigate the contradictions of physics he wanted to redress with his special theory of relativity (STR).

Imagine the waiter in the dining car of a train, moving at 100 km/h on a long straight stretch of rails. The waiter moves at 5 km/h in the dining car both in and against the direction of travel of the train. At which speed does he actually travel?

We make two observations: First of all it is obvious that speeds are relative and not absolute. They always refer to a given coordinate system. We have the choice of fixing our coordinate system to the dining car or to the ties of the railway track. If we sit 'in peace' in the dining car, then the waiter moves forwards and back at  $\pm 5$  km/h.

It was already clear to Galileo and Descartes, how fast the waiter moved in a reference system in which the rails are at rest and on which the train moves with 100 km/h. The speeds of the train and that of the waiter relative to the train are simply added: he moves with 105 km/h or with 95 km/h, always in the direction of the train's velocity. Thus our second observation is that in Newtonian mechanics speeds simply add. (If the speeds are not parallel as in our example, then not only the signs, but also the directions must be considered, i.e., the velocities must be added as vectors.)

In **D1** we will formally prove the correctness of this speed addition within Newtonian mechanics. The proof shows beautifully, how in particular the idea of absolute time is presupposed.

#### Where then is the problem?

In 1856 the physicist James Clerk Maxwell successfully condensed the rich research results of Michael Faraday and others in the areas of electricity and magnetism into four formulas. In 1862 he published these in his paper "On Physical Lines of Forces" and in 1873 his masterpiece "A Treatise on Electricity and Magnetism" appeared in two volumes. Maxwell demonstrated in pure mathematical form the fact that electrical and magnetic fields propagate as waves in space. In 1886 Heinrich Hertz proved the existence of such electromagnetic waves experimentally.

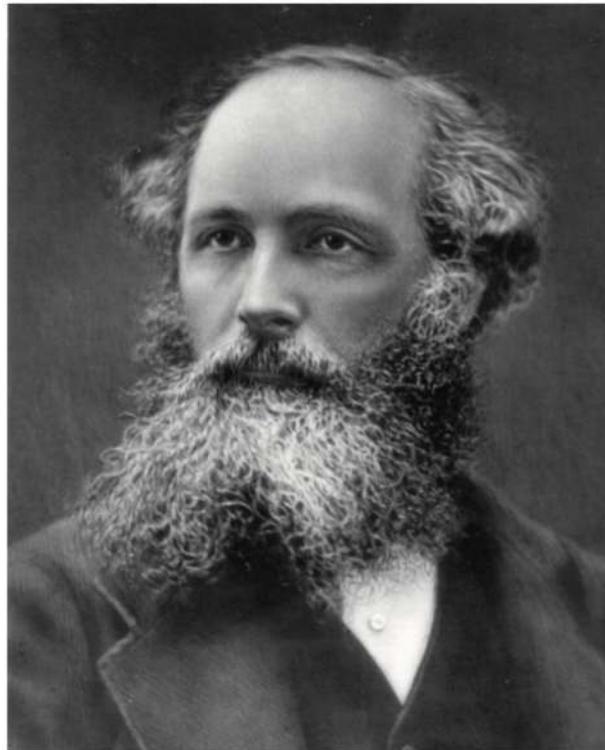
The propagation speed of these waves in a vacuum is given by the expression

$$c = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}}$$

where  $\epsilon_0$  is the electrical field constant, which for example also arises in the force law of Coulomb, and  $\mu_0$  is the appropriate magnetic field constant. Of course, Maxwell had already noticed that this value corresponds exactly to the speed of light in a vacuum (which by the way differs from that in air only very slightly). This implies however that the speed of light must also be a constant of nature, just as are the electrical and the magnetic field constants!

Thus this beautiful theory of Maxwell, which was distilled out of a large body of experimental work and which itself was afterwards splendidly confirmed, demonstrates that the speed of light in a vacuum is a constant of nature. If we accept that the relativity principle not only applies to mechanics, then it must also be true that Maxwell's equations apply in any inertial frame, with the same values for the constants of nature. The speed of light would be a constant, whose value would be the same in every inertial frame. The speed of the forward shining light of a forward moving locomotive must be exactly equal to that of one at rest or even one moving backwards! The speed of light is thus independent of the movement of its source. This however contradicts the vector addition of speeds, which we have also presented as fact within Newtonian mechanics.

Seen as a package Newtonian mechanics, the relativity principle and Maxwell's theory of electromagnetism are incompatible!



James Clerk Maxwell (1831 - 1879)

In order to describe the dilemma more clearly, we introduce the following abbreviations:

- N** Newtonian mechanics with absolute time and absolute space
- R** General relativity principle: All inertial frames are equal
- M** Maxwell's theory of electromagnetism

One cannot have **N**, **R** and **M** at the same time. From **N** and **R** follows the addition of velocities which implies that the light of the forward moving locomotive moves in the rail track's inertial frame at  $c + 100 \text{ km/h}$ . This does not fit **M**. From **M** and **R** follows the constancy of the speed of light which implies that the light of the forward moving locomotive has in each inertial frame the speed of  $c$ , the measured value is independent of the movement of the source and the receiver!

### Which possibilities remain?

One can keep **N** and limit **R** to the laws of **N**. In this case the beautiful equations of **M** apply unchanged only if the reference frame in Newton's absolute space is at rest, and other inertial frames are adapted. Ugly!

It amounts to almost the same, if one holds to **N** and **M** and abandons **R**. Now the experimenter has the additional task of determining his speed in absolute space, where the propagation medium of electromagnetic waves, the so-called ether, resides. This task was accepted by A. Michelson and E. Morley, following a suggestion by Maxwell.

If one tries to keep **R** and **M** then he must supply an 'improved' version of **N**! Before Einstein nobody had the courage to consistently pursue this path. Nevertheless Einstein could profit from many predecessors: FitzGerald and Lorentz suggested a formula for 'the contraction' of the measuring apparatus in the direction of motion of the earth through the ether. Also Lorentz had already around 1900 briefly introduced a 'local time', in order to explain the results of certain experiments. The great mathematician Poincaré coined in 1905 the expression 'Lorentz transformations' and (at the same time and independently of Einstein) showed that these transformations form a mathematical group and that **M** under such transformations is invariant. The fruit was thus ripe for the plucking (for details of the history of the STR refer to chapter 6 in [10]).

One could even claim that it was aesthetic reasons that actually induced Einstein to keep **R** and **M**. The introduction of his famous article of 1905 begins as follows:

"It is well known that Maxwell's electrodynamics - as usually understood at present - when applied to moving bodies, leads to asymmetries that do not seem to be inherent in the phenomena. Take, for example, the electrodynamic interaction between a magnet and a conductor. The observable phenomenon here depends only on the relative motion of conductor and magnet, whereas the customary view draws a sharp distinction between the two cases, in which either the one or the other of the two bodies is in motion." [09-123]

This article does not presuppose (at least in the first part) any higher knowledge in mathematics and is very much recommended to the reader.



Einstein at his desk in the patent office in Bern 1902

## A4 Einstein Cuts the Gordian Knot

Einstein declared **R** and **M** to be valid without any restrictions and showed how one must modify **N**, so that everything fits together without contradiction. He placed at the beginning of his theory, which today we call special theory of relativity (STR), the following postulate:

- R** All inertial frames are equal. The laws of nature, including the values of the constants arising therein, are the same in all inertial frames
- M** Maxwell's electrodynamics is valid without restrictions

In the last section we saw that **R** and **M** imply that the (vacuum) speed of light  $c$  has the same value in all inertial frames and is therefore a constant of nature. In the usual mks system of units this value is

$$c = 299,792,458 \text{ m/s}$$

The equals sign is correct: Since 1983 the meter is no longer a fundamental value, but is defined by this value of  $c$  and the second! Thus today the STR is the basis even for the definition of our basic metrics! The absoluteness of the speed of light in the STR firmly couples space and time together. A distance of approximately 300,000 km length corresponds to one second of time.

The boldness of Einstein's idea is oftly praised in today's text books. So do Roman Sexl, Ivo Raab and Ernst Streeruwitz in their very recommendable physics book for high school students (translation by Samuel Edelstein):

"Finally in the year 1905 a formerly unknown technician of the Swiss federal patent office in Berne stepped into the public eye with a new idea. His name was Albert Einstein, and his article 'On the Electrodynamics of Moving Bodies' proceeded from the idea that perhaps one could not measure the earth's movement through the ether because the ether does not exist!"  
[11-10]

Wilhelm Wien suggested in 1912 to award the Nobel Prize for physics in equal parts to Lorentz and Einstein:

"From a purely logical point of view, the relativity principle must be considered as one of the most significant accomplishments ever achieved in theoretical physics. ... [Relativity] was discovered in an inductive way, after all attempts to detect absolute motion had failed. ... While Lorentz must be considered as the first to have found the mathematical content of the relativity principle, Einstein succeeded in reducing it to a simple principle. One should therefore assess the merits of both investigators as being comparable. ... " [10-153]

Einstein would have agreed with this. He himself had designated only one of the 5 works of his 'annus mirabilis' as 'quite revolutionary', namely the one concerning the photoelectric effect! When Einstein in 1921 finally received his long overdue Nobel Prize, the reason also specifically emphasized that work. Einstein always spoke of Lorentz with the greatest respect. All 5 works were by the way published, along with good introductions, by John Stachel in [09].



Albert Einstein and Hendrik Antoon Lorentz (1921)

In the STR one can also replace the postulate **M** with the special requirement that  $c$  is a universal constant. In the following chapters we will demonstrate in detail, how one can derive the STR from the relativity principle and the constancy of  $c$ . Fortunately, a complete presentation of the STR requires only modest mathematical knowledge. We will not have to struggle like Kepler, who writes in his introduction to the "New Astronomy":

"I, who consider myself a mathematician, quickly fatigue the power of my brain through the re-reading of my work, in an attempt to recognize the sense of the proofs, which I indeed originally inserted with my own understanding into the diagrams and the text, and from which diagrams I again want to glean an understanding. If I obviate the heavy comprehensibility of the material by interspersing detailed descriptions then I appear to be garrulous in mathematical things and that is the opposite mistake." (translation by Samuel Edelstein from [08-19])

## A5 Suggestions

In order to keep this book to a manageable size, much that would otherwise be interesting to include, must be omitted. I would like to draw my reader's attention to a part of this material with these 'Suggestions'.

1. Read biographies! Copernicus, Kepler, Galileo and many others did not only make large contributions, but were also interesting people.
2. Read original publications or at least parts of them. The fundamental physical concepts are usually not hidden in cryptic mathematics. Copernicus, Kepler, Galileo and Newton are often a pleasure to read.
3. Read the introduction to other books about STR. These usually begin with the problem of the ether and the attempts of Michelson and Morley to measure the earth's movement through the ether. You now know the principle ideas and do not run the risk of getting lost in the details.
4. Sigmund Freud's book "The Interpretation of Dreams" appeared in 1900. It marks the parallel break-down of a supposed certainty in a completely different field.
5. Likewise around 1900 Munch and others begin to contrast themselves to the 'beautiful' impressionists and in music Ravel and others begin experimenting with musical modalities.
6. In 1899 Hilbert published "The Foundations of Geometry". This led to the attempt to prove the consistency and completeness of arithmetic. Kurt Gödel showed however in 1930 that this was illusory.
7. At the same time Russel and Whitehead were looking at the lack of logical underpinnings in set theory and in 1905/10 in their tome 'Principia Mathematica' tried to place logic and set theory on a proper foundation.
8. In 1900 Planck published his derivation of the radiation law and introduced thereby the idea of the quantization of energy. The truly revolutionary aspect however was first revealed with the article by Einstein in 1905 on the photoelectric effect ('On a Heuristic Viewpoint Concerning the Production and Transformation of Light').
9. Study the behavior of a partly filled glass in a moving train or on a revolving turntable. High school mathematics is sufficient to derive the shape of the surface in relation to the acceleration or the angular speed. Read the section in Newton's 'Principia' concerning his bucket experiment.
10. Travel up and down on a bathroom scales in an elevator.
11. Consider Foucault's pendulum experiment concerning the place of execution: a) at the north pole; b) at the equator; and c) at middle latitudes. It shows that the earth rotates 'absolutely'.
12. What would the result of the pendulum experiment be according to Foucault, if there was only the earth and otherwise no other heavenly bodies in the universe?



Kurt Gödel and Albert Einstein in Princeton 1954



Spiral Galaxy Messier 83 (VLT ANTU + FORS1)

ESO PR Photo 41/99 (29 November 1999)

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## B The Three Fundamental Consequences

Adhering to the principle of relativity and Maxwell's equations has three fundamental consequences: The simultaneousness of two events loses their absolute character, different observers of an activity will no longer measure the same duration, and the distance between two points (or the length of an object) loses its absoluteness. We discuss the experimental confirmation of these basic phenomena from two perspectives. Finally we derive the amount the two clocks appear desynchronized to a moving observer, given that they are synchronized in their own inertial frame.

## B1 Primo: The Relativity of Simultaneity

Much ink has been spilled concerning the nature of 'time'. Einstein's break-through insight however sounds quite banal: Time is what one reads from a nearby clock:

### Time is what one reads from a local clock

Deep insights are often not at first sight perceptible as such ...

First we want to convince ourselves that it is possible to synchronize several identical clocks which are at rest in an inertial frame at different places. Often the following method is suggested: two clocks are at points A and B respectively. A flash of light is released at the midpoint of AB and on arrival of the light each clock is set to 0000 and started.

The problem is: how does one synchronize a third clock C with clock A without losing the synchronization between A and B? And isn't finding the midpoint already a problem? This 'standard method' is actually unworkable.

It is however quite possible to synchronize as many clocks as desired with a given clock A: The 'master' clock A emits a flash of light at an arbitrary but well-known time  $t_0$ . As soon as the light arrives at clock B, it is firstly reflected, secondly B's clock is set to 0000 and thirdly it is started. Clock A records time  $t_1$ , when the light reflected from B arrives again at A. One calculates the elapsed time  $(t_1 - t_0) / 2$  for the light from A to B, records the value  $t_0 + (t_1 - t_0) / 2$  and sends it by snail mail to B. The (continuously running) clock B is then *advanced* by this value. One does not need the midpoint AB at all and in addition one obtains the distance between the two clocks.

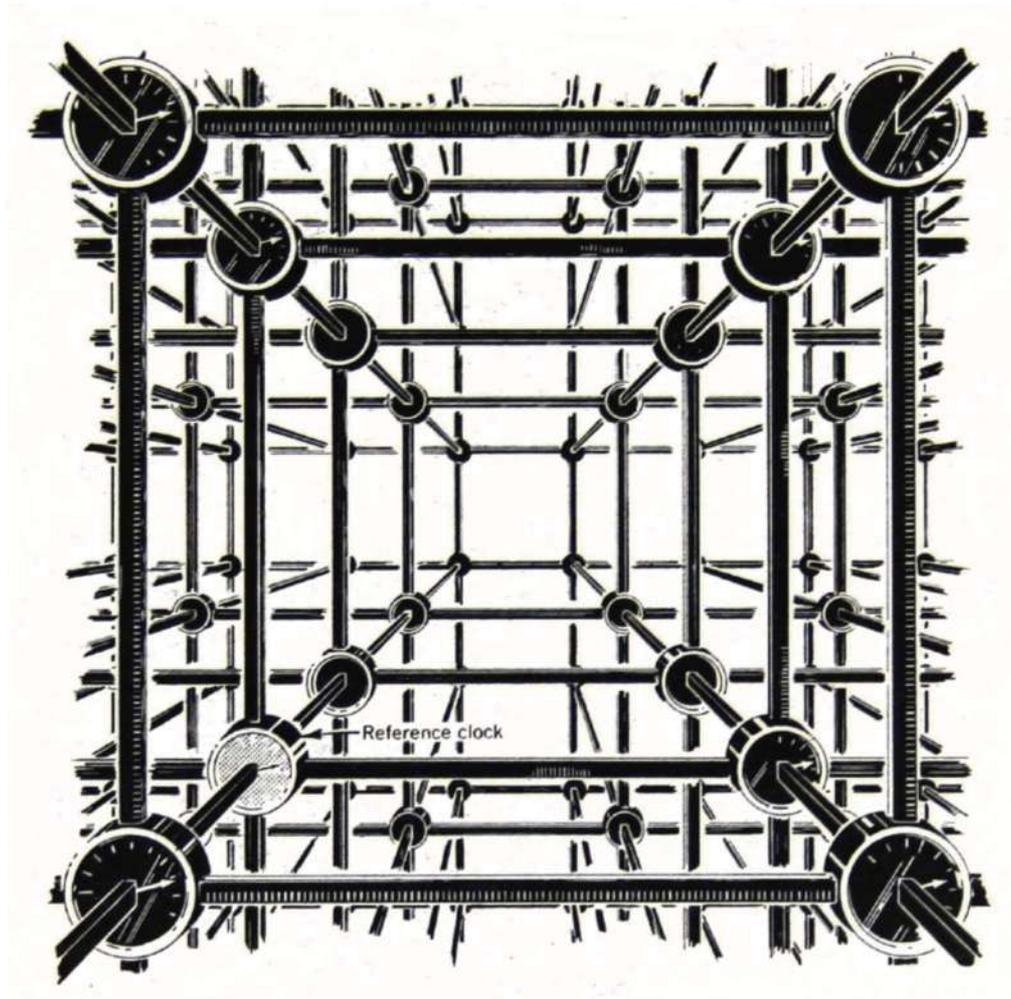
Hans Reichenbach pointed out in different publications starting from 1920 that this definition implies a further assumption, i.e. the isotropy of space (a collection of Reichenbach's early writings on space, time and motion in English translation has been edited by Steven Gimbel and Anke Walz in [12]). In particular the speed of light should be equal in all directions. Measuring the one-way speed of light presupposes distant clocks which have already been synchronized. Therefore the synchronization of distant clocks and the measuring of the one-way speed of light have a circular relationship to each other. When we computed the elapsed time for the light from A to B as  $(t_1 - t_0) / 2$  we tacitly assumed that the light needs equal time to travel in both directions! The postulate of isotropy was hidden in this assumption. The book "Concepts of Simultaneity" by Max Jammer [13-218] presents two simple axioms which a set of clocks must meet, in order to be synchronizable. The formulation of the first axiom is ours:

1. If a clock A sends out two light signals with  $\Delta t_A$  time difference, then each further clock B must receive the signals with  $\Delta t_B$  time difference, where  $\Delta t_B = \Delta t_A$ .
2. The time required for light to traverse a triangle is independent of the direction taken around the triangle.

The first axiom must surely be fulfilled if synchronized clocks are to remain synchronized. Obviously it can be fulfilled only by clocks which are at rest relative to each other! The second axiom (called the "round trip axiom") guarantees that the speed of light is independent of direction. Taken together the two axioms are necessary and sufficient so that a set of clocks can be synchronized.

Since we will need the postulate of isotropy of space in **B3**, we introduce it here to the STR. Its operational formulation concerning the speed of light is found in the "round trip" axiom.

Thus, in an inertial frame one can have clocks at arbitrary locations, which are all synchronized in the sense described above. Measuring the point of time of an event means to read the time from such a synchronized clock positioned at the location of the event. Thus we now have a conception in terms of a hardware view of an inertial frame, as represented in [14-37]:



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What does one see in this scaffolding of clocks on the dial of one of the distant clocks?

For all physical quantities the following three aspects can never be completely disassociated:

- a) the definition of the quantity
- b) the method of measurement of this quantity and
- c) the definition of its units of measurement.

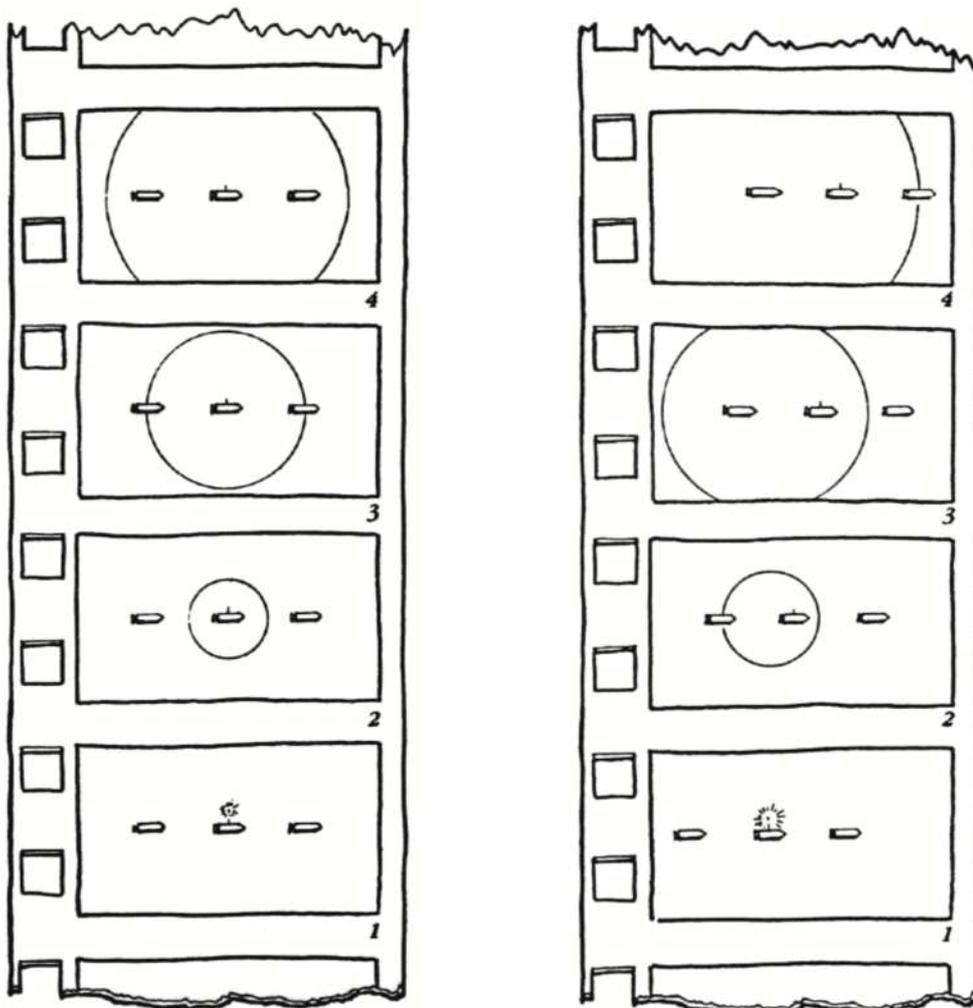
To define 'time' as a physical quantity means to construct some copies of an accurate clock and to define how to compare the times indicated by these clocks (consult the Wiki articles for the second and for the international atomic time !).

Einstein's insightful contribution to the development of the STR was to show that this operational approach eliminates all difficulties.

Thus in an inertial frame we can get along with one single time. We are allowed to speak of *the* time  $t$  in an inertial frame. However, different inertial frames usually have different chronologies for events. Epstein demonstrates this very beautifully in [15-33ff]. He considers an interstellar fleet of three spaceships, which travel in a row in space at a constant distance from each other:



There is an inertial frame, in which the three spaceships are at rest. A radio call from the flagship will reach the other two spaceships at the same time in this inertial frame (movie strip on the left, should be read from the bottom to the top). On the other hand, for a viewer in an inertial frame in which the fleet moves, the radio call reaches the leading spaceship later than it does the following ship (movie strip on the right)! This is a direct consequence of the fact that the radio signal in each inertial frame travels in all directions with the constant speed  $c$ .



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Quantitatively however the strip on the right is not completely correctly drawn. Here  $c$  appears to be somewhat larger than for the one on the left, and above all the fleet does not move with uniform speed, which the reader can easily verify with a ruler.

Given the postulate that  $c$  has the same constant value in every coordinate system and that light or radio signals spread in each inertial frame with the same speed into all directions in space, it follows immediately that it makes sense only within an inertial frame to say that two events take place at the same time. Poincaré had already pointed out in 1902 :

"The English teach mechanics as an experimental science; on the Continent it is taught always more or less as a deductive and a priori science. The English are right, no doubt. How is it that the other method has been persisted in for so long; how is it that Continental scientists who have tried to escape from the practice of their predecessors have in most cases been unsuccessful? On the other hand, if the principles of mechanics are only of experimental origin, are they not merely approximate and provisory? May we not be some day compelled by new experiments to modify or even to abandon them? These are the questions which naturally arise, and the difficulty of solution is largely due to the fact that treatises on mechanics do not clearly distinguish between what is experiment, what is mathematical reasoning, what is convention, and what is hypothesis. This is not all.

1. There is no absolute space, and we only conceive of relative motion; and yet in most cases mechanical facts are enunciated as if there is an absolute space to which they can be referred.

2. There is no absolute time. When we say that two periods are equal, the statement has no meaning, and can only acquire a meaning by a convention.

3. Not only have we no direct intuition of the equality of two periods, but we have not even direct intuition of the simultaneity of two events occurring in two different places. I have explained this in an article entitled "La Mesure du Temps".

4. Finally, is not our Euclidean geometry in itself only a kind of convention of language? Mechanical facts might be enunciated with reference to a non-Euclidean space which would be less convenient but quite as legitimate as our ordinary space; the enunciation would become more complicated, but it still would be possible.

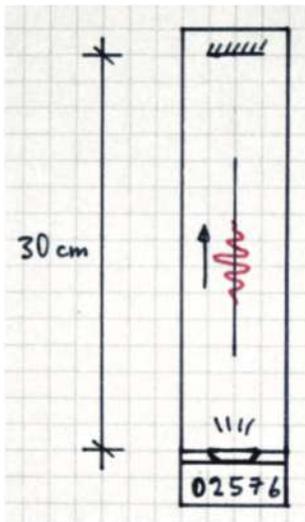
Thus, absolute space, absolute time, and even geometry are not conditions which are imposed on mechanics. All these things no more existed before mechanics than the French language can be logically said to have existed before the truths which are expressed in French. We might endeavour to enunciate the fundamental law of mechanics in a language independent of all these conventions; and no doubt we should in this way get a clearer idea of those laws in themselves." [16-89ff]

Einstein and his friends Solovine and Habicht (from right to left) carefully studied Poincaré's book at the "Akademie Olympia". With his operational definitions Einstein analyzed the "conventions", which permit us to speak about simultaneousness within an inertial frame. Just as clearly he showed that clocks which are synchronized in one frame do not run synchronously when observed from another moving frame. Also the amount of desynchronization was given an exact numerical value. We address this quantitative aspect in **B6**.



## B2 Secundo: Fast Clocks Tick More Slowly

It gets even worse: not only does the concept of simultaneousness become meaningless when we observe events from two separate inertial frames which are moving with respect to each other, but the river of time itself flows at different speeds! For the derivation of this difference we need only the Pythagorean Theorem.

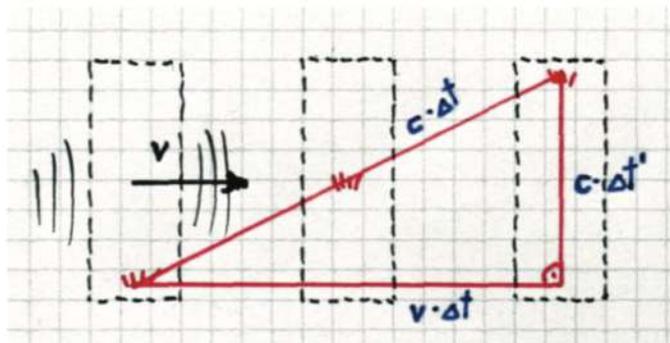


We assume the constancy and universality of  $c$  and consider “light clocks” with the following design:

In a pipe a flash of light travels from the bottom up to a mirror at the top, where it is reflected (“tick”) back to the bottom. There it is sensed by a photoelectric cell (“tock”) releasing a new flash and incrementing a counter by 2. The counter can be read at any time. Take a moment and consider why the clock should be 30 cm in length and why the counter is increased each time by 2.

Imagine that we have several such clocks and that some are set up and synchronized along the  $x$  axis of our coordinate system at known distances. A further identically constructed clock moves with speed  $v$  past these “resting” clocks (only this “fast” clock is shown below at three positions in the diagram!). How much time elapses in the resting system, during the time the “fast” clock makes a single “tick”?

The distance light travels in the moving clock (call it the prime system with measured time  $t'$ ) is 30 cm or, in general,  $c \cdot \Delta t'$ . But what is the distance this light travels as seen from the resting system (call it the non-prime system with measured time  $t$ ) and in relation to which the moving clock travels along the  $x$ -axis with velocity  $v$ ? Given the constancy of the speed of light this will be, of course,  $c \cdot \Delta t$ . These two distances are however not equal and thus the time intervals,  $\Delta t$  and  $\Delta t'$ , must differ! The Pythagorean Theorem provides us with the relationship between these two values:



Obviously more time elapsed in the resting, non-prime-system, than in the moving, prime-system, since the light travelled a longer distance in that system. Thus:

$$(c \cdot \Delta t)^2 = (v \cdot \Delta t)^2 + (c \cdot \Delta t')^2 ; c^2 \cdot (\Delta t)^2 = v^2 \cdot (\Delta t)^2 + c^2 \cdot (\Delta t')^2 ; (\Delta t')^2 = (\Delta t)^2 \cdot (1 - v^2/c^2)$$

and we get

$$\Delta t' = \Delta t \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

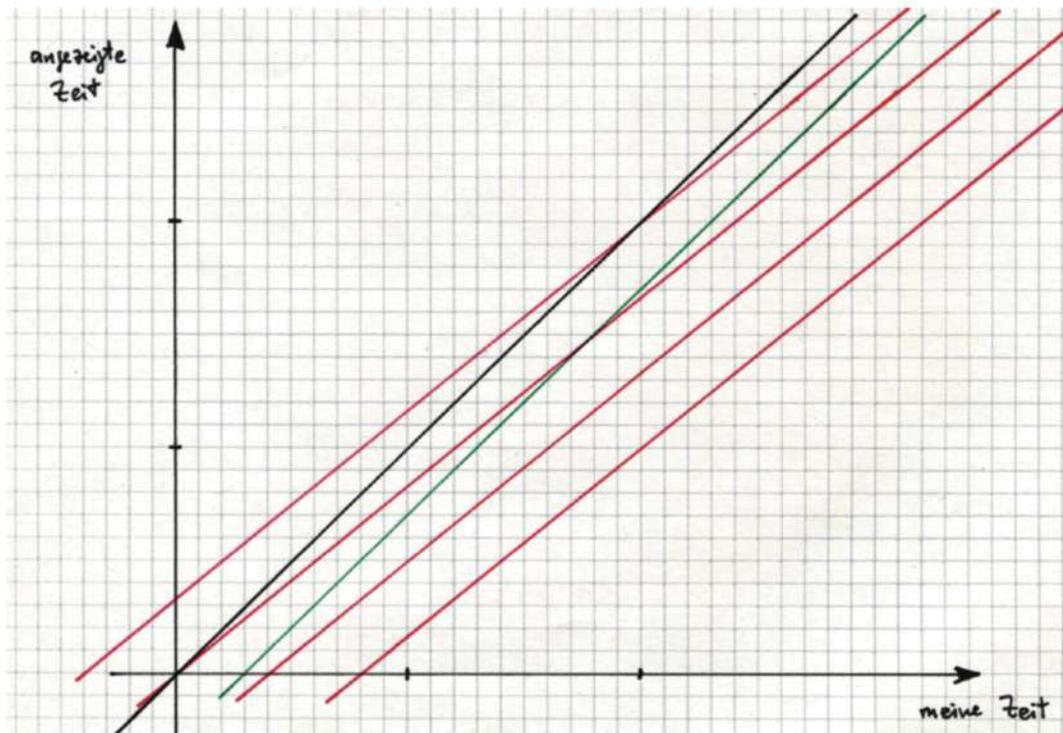
Thus moving clocks tick more slowly, compared to a set of clocks at rest. One calls this effect 'time dilation'. Search the Internet with keywords 'light clock' or 'time dilation' and you will find innumerable nice animations concerning this consequence of holding to **M** and **R**.

In the prime system we measure the *proper time* of a "tick". A proper time interval is measured by a single clock between two events that occur at the same place as the clock. In the non-prime system we need two synchronized clocks to measure the time interval corresponding to a single "tick" of the moving clock.

The proper time interval is always the *longest* time interval measured by a single clock between two events, that mark the beginning and the end of a given process! Section **H4** is entitled with "The Principle of Maximum Proper Time (Eigenzeit)". From the point of view of the prime system, the non-prime clocks are moving, and all of them tick more slowly than its own clock which is at rest. So each of those clocks would measure a shorter time interval for the same process. How do we explain however that we just measured a longer duration in the non-prime system? Does this not contradict our principle of maximum proper time? Perhaps you already see how to resolve this apparent contradiction: The important fact is that in order to make the measurement in the non-prime system, we need at least two distant clocks...

We will clarify this point completely towards the end of section **B6**!

Thus little remains of Newton's absolute time! It makes sense to draw a diagram showing the time indicated by identical, perfect (!) clocks in relationship to the time indicated on my perfect clock:

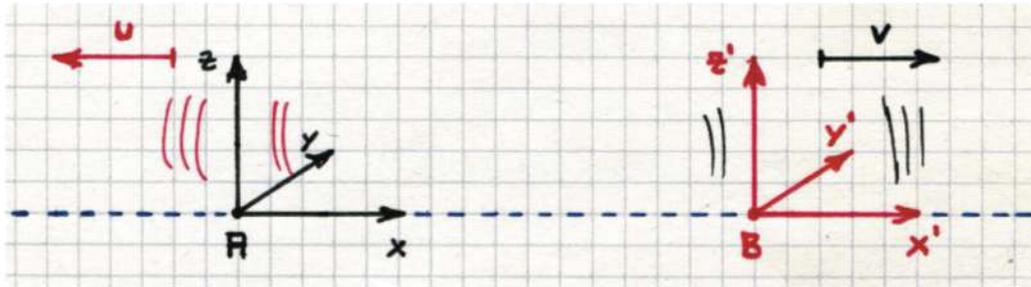


black: my clock and all other perfect clocks synchronized with it in my inertial frame  
green: a correct, but badly synchronized clock at rest in my inertial frame  
red: "fast" clocks, which can be synchronized in their own inertial frame

Try adding to the diagram an imperfect, but more or less well synchronized clock at rest. Also add a second clock at rest, which is synchronized at time point 10, but then runs fast!

### B3 Tertio: Moving Yardsticks Are Shorter

The inertial frame B (prime, red) moves with constant speed of  $v$  along the  $x$  axis of the inertial frame A (non-prime, black):



The STR should be consistent in the following sense: both A and B make the same statements about which time intervals or lengths in A and B are measured. They will not measure the same values, but they both can figure out, what the other one measured, and agree about these values. We draw from this fact the following important conclusion: If B moves for A with velocity  $v$  in the positive  $x$ -direction, then A moves for B with the velocity  $-v$  in the  $x'$ -direction! In addition to the speed of light  $c$  both also have the amount of their relative velocity in common. Most authors assume that this is self-evident. Is it really?

We consider what alternatives might be possible: Assume that B measures a relative velocity  $u$  of the two systems where  $|u| < |v|$ . In this case A also knows that B measures a smaller relative velocity. If space is isotropic (looks the same in all directions) and the STR is consistent as described above, then the situation is perfectly symmetrical. In this case B will correspondingly state that A measures a smaller relative velocity. Thus we have a contradiction: it follows for the magnitudes of the relative velocities that  $v < u < v$ , which is impossible. Therefore B can measure neither a smaller nor larger relative speed of the two systems than A, it must be that  $u = -v$  and  $|u| = |v|$ . Here again (remember **B1** !) we need the postulate of isotropy!

System A has 2 synchronized clocks at a distance  $\Delta x$  from each other. B moves with relative velocity  $v$  over this distance and measures with its clock the time  $\Delta t'$ , which elapses between the meetings with the two clocks of A. B uses  $\Delta t'$  and  $v$  to determine the distance between the two clocks in system A:  $\Delta x' = v \cdot \Delta t'$ .

But what does A observe? A measures  $\Delta t$  between the two clock meetings and the distance  $\Delta x$  of its clocks and determines the speed  $v$  of B:  $v = \Delta x / \Delta t$ . The value of  $v$  is the same for A and B, and so we have the following equation

$$\frac{\Delta x}{\Delta t} = v = \frac{\Delta x'}{\Delta t'} \quad \text{and thus} \quad \Delta x' = \Delta x \cdot \frac{\Delta t'}{\Delta t} = \Delta x \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

using the result of the last section. The distance  $\Delta x$  which is at rest in the system A appears in system B to be shortened by the same factor we have already encountered.

We have preferentially treated the  $x$ -direction (which corresponds to the  $x'$ -direction and the direction of the relative velocity); actually we know only that lengths of moving objects in the direction of relative motion are shortened. What is the behavior in perpendicular directions?

Consider Einstein's train, moving with speed  $v = 0.6 \cdot c$  in the x-direction on a long straight railroad line. If the train has the length 300 m in its own reference system, then we will measure it at a shortened length of 240 m (do the math!). Did the train also become narrower? If so then it would have fallen between the rails (which are at rest) when it reached a certain speed. That would, in principle, be possible. However, if moving objects would contract themselves perpendicular to the direction of motion, then from the point of view of the travelers in the train it would mean that the separation between the moving rail tracks became smaller! And the theory would require the track width to be too large and too small at the same time - impossible! Therefore we conclude that there is no 'lateral contraction'.

In summary:

Moving objects appear shortened in their direction of motion, that is, the length of an object measured at rest is always the longest. We call this the principle of *maximum proper length*. (Note: curiously google comes up empty in a search for this expression whereas the principle of maximum proper time is well known.) Perpendicular to the direction of the relative motion the measured values of all observers agree. The following formulas apply:

$$\Delta x' = \Delta x \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

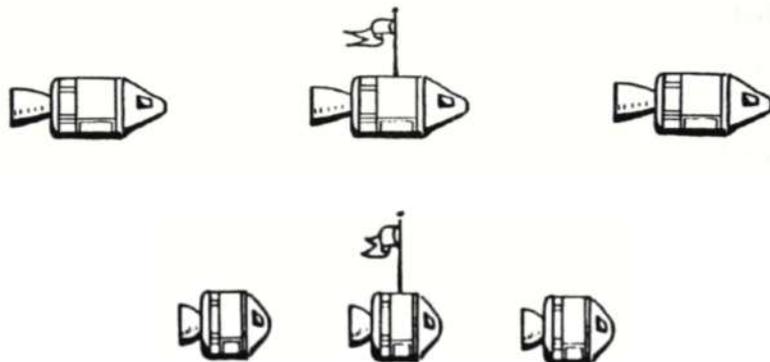
$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

In addition the value of the radical agrees for both reference systems, since in both the square of the relative velocity has the same value.

By the way, the absence of lateral contraction is quite significant for our argument in **B2**! Otherwise the path of the light perpendicular to  $v$  would not be of equal length in both systems, and we would not have a unique length of the vertical leg of the triangle. The perpendicularly standing light clock only becomes more narrow and not shorter or longer. We were indeed fortunate...

Thus Epstein's small fleet, if at first it is at rest and then speeds past an observer at very high speed, appears so (Illustrations [15-39f]):



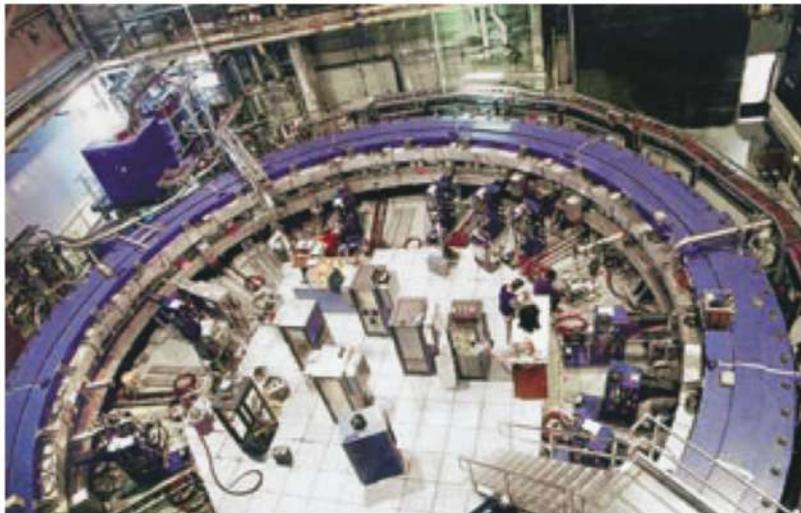
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## B4 Muons 1: An Experimental Confirmation

For a long time there existed hardly any experimental confirmation to Einstein's relativity theories. This changed dramatically with the development of atomic clocks and modern electronics starting around 1960, providing a boost to relativity theory as an area of research.

A first confirmation of relativistic time dilation was found in the extended half-life of fast-moving muons (B.Rossi and D.B.Hall 1941). These come into being at a height of 10 to 20 km over the earth's surface, when high-energy particles of cosmic radiation strike atoms in the terrestrial atmosphere. Muons differ from electrons in that they have a much larger mass and are unstable. Slow muons have an average life span or 'half-life' of  $1.52 \mu\text{s}$ . The extremely fast muons, which are produced by the cosmic radiation, move nearly at the speed of light and should therefore according to Newton travel about  $1.52 \cdot 10^{-6} \cdot 3 \cdot 10^8 \text{ m}$ , approximately 456 m, during their life time. The flow of muons should therefore be halved, if one decreases the altitude by 456 m. However it actually decreases much more slowly. Since the muons are created at an altitude of about 15 km, they must travel 33 times the distance 456 m to reach sea level. Out of  $2^{33}$  muons only one should reach our detector. However this does not fit the observed density of the muon stream: in Germany approximately 200 muons per square meter per second are counted at sea level.

CERN scientists tested time dilation of muons quantitatively much more exactly in 1975. Large quantities of muons were produced and captured with 99.942% the speed of light in a special storage ring. It showed that their half-life at this speed amounts to  $44,6 \mu\text{s}$ , in complete agreement with our formula of **B2** (do the math!). More details are provided in [11-13f].



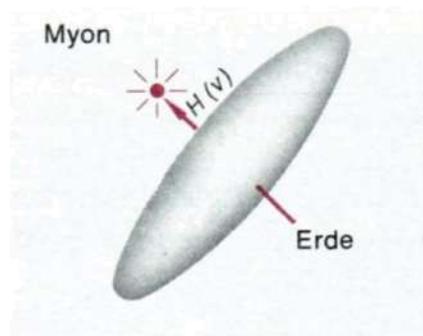
Muon storage ring at the Brookhaven National Laboratory, USA

## B5 Muons 2: A Second Look

Let's again consider the muons produced at an altitude of 15 km by cosmic radiation. Such a muon is at rest in its own inertial frame and thus has the usual half-life of  $1.52 \mu\text{s}$ . This small half-life should only be enough for the earth to approach the muon by around 456 m. Why then do so many muons experience the arrival of the earth's surface?

Time dilation is no help here. But the 15 km thickness of the atmosphere extends in the direction of the earth's motion towards the muon and thus appears to the muon 'Lorentz contracted'. Using the formula of **B3** we calculate for  $v = 0.99942 \cdot c$  a root factor of 0.03405. For our muon the 15 km diminishes to 511 m! The earth traverses these 511 m in somewhat more time than the half-life of a muon, i.e. nearly half of all muons experience the arrival of the earth's surface.

Seen by the muon the earth keeps its old cross sectional area. The earth's diameter and also the thickness of its atmosphere shrink however in the direction of motion down to 3.4% of the rest value. The earth thus takes on the shape of a flat disk. [11-14f] offers no derivation of the length contraction, but it indicates in the text and in one of the many border illustrations (which make the book so attractive) that this contraction offers an explanation in the muon's inertial frame. The border illustration on [11-15], shown below, contains however 2 errors, a harmless one in the picture and a worse one in the text. Can you find the two errors?



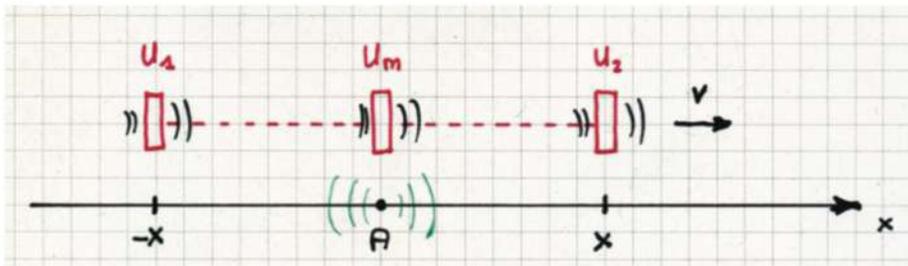
*Seen from the point of view of the muons, the earth which is approaching them at nearly the speed of light, appears greatly flattened. All distances in the direction of motion are shortened. The muons are thus able to travel the distance to the earth's surface within their life time.  
(translation by Samuel Edelstein)*

In both representations (earth at rest with moving muons and muon at rest with approaching earth) one arrives at the same conclusion about the portion of muons which collide with the earth's surface. The reasoning is however completely different. The 'history' each tells differs strongly from that of the other one. 'History' is truly 'his story'... (wordplay personally communicated to me by Floyd Westermann). This however results in no contradictions or conflicts concerning the physics.

## B6 Quantitative Aspects of the Relativity of Simultaneity

In **B1** we concluded that the synchronization of a set of clocks in different inertial frames must fail. Even if the synchronization were possible for a given time point it would make little sense considering **B2**. It is however possible to exactly say by how much 2 clocks which are synchronized in the red system B are desynchronized from the point of view of the black system A. We will now derive this formula. It appears only in a few books concerning STR. It is however indispensable, if one really wants to put all the pieces together. This will be clear from the set of example problems presented at the end of this section.

We introduce three clocks  $U_1$ ,  $U_m$  and  $U_2$  moving in relationship to each other like Epstein's small fleet, that is, at constant distance from each other with velocity  $v$  in the  $x$ -direction of the black, at rest, non-prime system A.  $x'$  is the distance of neighboring clocks in the red, fast-moving, prime system B;  $x$  is the corresponding value measured in the black system.  $x'$  is larger than  $x$ .



At the exact point when  $U_m$  flies past the zero point A of the black system, a flash of light is released. We call this time point 0. In the red system the two clocks  $U_1$  and  $U_2$  are thereby synchronized ( $U_m$  remains in the middle of  $U_1$  and  $U_2$  whether or not Lorentz contraction takes place!). However when are  $U_1$  and  $U_2$  for the black system triggered by this flash of light?

$U_1$  is flying toward the flash;  $U_1$  will encounter the flash at time point  $t_1$  yielding  $t_1 \cdot c = x - t_1 \cdot v$ ; and thus  $t_1 = x / (c + v)$

$U_2$  is flying away from the flash;  $U_2$  will encounter the flash at time point  $t_2$  yielding  $t_2 \cdot c = x + t_2 \cdot v$ ; and thus  $t_2 = x / (c - v)$

The clock in front,  $U_2$ , will be started for the black system with the following delay:

$$t_1 - t_2 = \dots \text{ (do the math!) } \dots = -2 \cdot v \cdot x / (c^2 - v^2)$$

That is the time difference for the black system, which knows however that the red clocks run more slowly than its own. We discover the time difference of the red clocks only if we multiply this value by our radical:

$$\Delta t' = (t_1 - t_2) \cdot \sqrt{\dots} \text{ (do the math!) } \dots = -2 \cdot x \cdot (v/c^2) / \sqrt{\dots}$$

$2 \cdot x / \sqrt{\dots}$  is exactly the distance  $\Delta x'$  of the clocks  $U_1$  and  $U_2$  as measured in their own system! Thus we have the quite simple result

$$\Delta t' = -\Delta x' \cdot \frac{v}{c^2}$$

The red clocks are synchronized in their own system and have distance  $\Delta x'$  in the direction of the relative motion. However, these clocks are desynchronized in the black system by the amount  $\Delta t'$ . One can write the result differently, seeing more clearly that the formula is as simple as possible:

$$\Delta t' \cdot c = -\Delta x' \cdot \frac{v}{c}$$

The factor  $c$  on the left of the equals sign serves only to convert times into lengths. The de-synchronization is thus proportional to the distance between the red clocks in the direction of motion and also to the ratio  $v/c$ .

That one cannot do without this formula, if one wants to present the whole situation without contradiction, will be clarified through a careful study of the following

### Sample Problem

A particle moves with  $v = 0.8 \cdot c$  through a 12 m long pipe, which is equipped with detectors at both ends, which in turn contain clocks, allowing one to measure the transit flight time precisely. The pipe is at rest in the black system. Let the particle's rest system be the red system. We pose and then answer the following questions:

1. How long does the transit flight of the particle through the pipe last for black?
2. How much time elapses thereby in the red system (of the particle) from the point of view of the black?
3. How long is the pipe for red?
4. How long does it last for red, until the pipe has raced over the particle?
5. How much time elapses from the point of view of red for this flyby on each clock of black?
6. How does red explain the measured value of black??

The questions 5 and 6 are omitted in most text books, whereby they form the capstone of understanding of the STR.

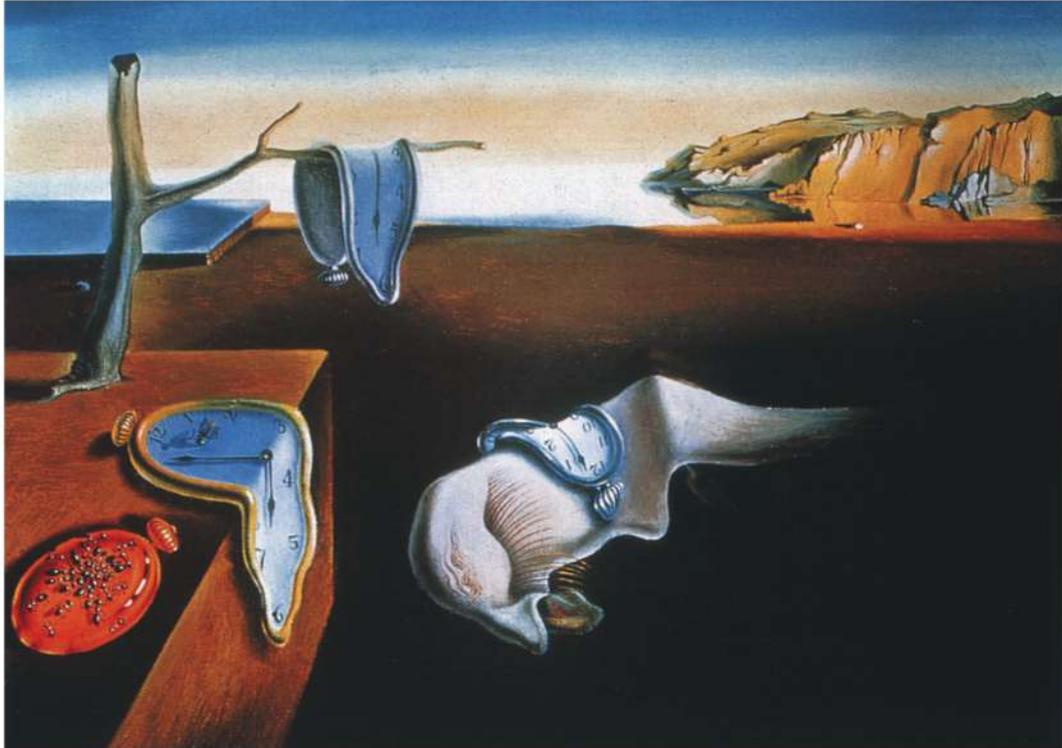
### The answers:

1. Time is distance divided by velocity:  $\Delta t = \Delta x/v = 12 \text{ m} / (0.8 \cdot 3 \cdot 10^8 \text{ m/s}) = 50 \text{ ns}$
2. Because of the time dilation red will measure a shorter duration:  
 $\Delta t' = \Delta t \cdot \sqrt{v} = 50 \text{ ns} \cdot 0.6 = 30 \text{ ns}$
3. Red sees the pipe as Lorentz-contracted:  $\Delta x' = \Delta x \cdot \sqrt{v} = 12 \text{ m} \cdot 0.6 = 7.2 \text{ m}$
4. Until the 7.2 m long pipe has completed flying over red:  
 $\Delta t' = \Delta x'/v = 7.2 \text{ m} / (0.8 \cdot 3 \cdot 10^8 \text{ m/s}) = 30 \text{ ns}$  (in complete agreement with black!)
5. The fast-moving clocks of black tick more slowly for red than they do in their own frame. Thus the flyby in the black system as seen by red lasts only  $\Delta t = \Delta t' \cdot \sqrt{v} = 30 \text{ ns} \cdot 0.6 = 18 \text{ ns}$  ( !! )
6. Also red knows that black measured 50 ns, however red attributes it to the fact that the two clocks of black are desynchronized by  $\Delta t = \Delta x \cdot v/c^2$  which numerically constitutes exactly  
 $12 \text{ m} \cdot 0.8 / (3 \cdot 10^8 \text{ m/s}) = 32 \text{ ns}$ . The 18 ns duration plus 32 ns desynchronization yield together the 50 ns that black measured with his two clocks!! Check that the sign is also correct.

## B7 Problems and Suggestions

1. Compute the root term for different values of  $v$ : For cars, airplanes, rockets etc. What does it mean for Newton's absolute time, when one considers only 'earthly' speeds?
2. How quickly does one have to move, in order that "one hour takes only 3599 s"?
3. How long would the running of '24 hours of Le Mans' last for the drivers, if they drive on average (somewhat exaggerated) at 324 km/h? The answer may depend on the pocket calculator that is used...
4. How far does light actually travel in a nanosecond? What distance corresponds to this precisely measurable time interval?
5. Provide a table of the distances of the planets from the sun, measured in 'light minute' units.
6. Two rockets fly past each other at  $0.6 \cdot c$ . A measures the length of the other rocket B to be 40 m. What is the rest length of the rocket B, and how much are the clocks at the tip and at the end of rocket B for A desynchronized, given that they are synchronized for B? And which of the two clocks is running fast for A?
7. How fast does a clock have to move, in order to halve its running-time?
8. Signal travel times: The singing of Mick Jagger is broadcast directly from the microphone to a radio listener 300 km away. The listener sits 6.8 m from the loudspeaker. How long does the radio signal travel through the 'ether'? How long do the acoustic waves need to travel from the loudspeaker to the ear of the listener? How long does it take for someone who is at the 'live' performance and sits 34 m from the loudspeakers to receive the acoustic waves?
9. What would actually happen with the length of an object, which moves with double the speed of light? And how slowly would such a fast-moving clock tick??
10. Yet another Pythagorean Triple: The root is also quite pretty if  $v/c$  takes the values  $5/13$  or  $12/13$  ... The pair  $3/5$  and  $4/5$  should already be familiar to you.
11. Again two rockets, flying past each other at high speed: A measures when flying by that the two rockets are the same length. What does B measure?  
a) A is the same length as B      b) A is longer than B      c) A is shorter than B ?
12. Are fast-moving clocks, which are synchronized in their system, **really** synchronized or not? The question is just as meaningful as asking about the season: Is it now **really** winter or summer? If you are not sure, then call your uncle in Australia...
13. (Challenging) Derive the length contraction from observing a fast-moving light clock that is lying on its side! The flash of light travels back and forth in the same direction as the clock is moving and thus the 'tick' and 'tock' are not equivalently long to an observer at rest...

We have now described the relativity of objective time measurement. That subjective time experience is 'malleable' is well-known. Salvador Dali's 'The Persistence of Memory' fits both perspectives quite well:



Einstein once illustrated the subjectivity of time experience in the following way:

“An hour sitting with a pretty girl on a park bench passes like a minute, but a minute sitting on a hot stove seems like an hour.” [17-247]



Globular Cluster 47 Tucanae  
(FORS/VLT)

## C Epstein's Simple Explanation

We are astonished at the very simple explanation, which Epstein provides in [15] for the three basic phenomena of the STR and which he calls his 'myth'. Simple to construct diagrams permit one to represent quantitatively correctly time dilation as well as length contraction and the desynchronization of fast-moving clocks. We solve our sample problem using these diagrams and summarize the results of **B** using Epstein's representation.

## C1 Epstein's Myth

In 1981 Lewis C. Epstein published the book [15] "Relativity Visualized". In this book Epstein introduces a truly new and easily interpreted depiction of relativity theory. It is often referenced, but rarely really taken seriously. Most text books continue to exclusively use Minkowski diagrams to depict relativistic phenomena from the point of view of two coordinate systems in relative motion to each other.

Hermann Minkowski began his famous 1908 speech to the 80th Meeting of German Natural Scientists and Physicians with the following words:

"The views of space and time, which I wish to lay before you, have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth, space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality. ... " [10-152]

Minkowski continued in this lecture to develop a consistently four-dimensional viewpoint of physical events, by combining the three space coordinates and time into a four-dimensional vector. In so doing he provided the STR in a geometrical manner a much more descriptive depiction - at least for mathematicians. Einstein joked at first that he no longer understood the STR, since the mathematicians took it up; however later it was clear to him that the new perspective provided a great step forward.

This is precisely the starting point of Epstein's "myth", on which he develops his representation. He says: It only seems to us that we live in a three-dimensional world, in which we make our small trips as a function of linear time. We actually all live in four-dimensional space-time. Our perception stems from the fact that we always move with the speed of light in this four-dimensional space-time - and that the direction in which we move, we call *time*, and the three others, which are perpendicular to it, we call *space* ! That we cannot perceive spatially the direction in which we are moving with the speed of light, as we do the others, is quite understandably due to Lorentz contraction. Epstein explains it this way:

"To understand the Special Theory of Relativity at the gut level, a good myth must be invented, and here it is.

Why can't you travel faster than light? **The reason you can't go faster than the speed of light is that you can't go slower. There is only one speed. Everything, including you, is always moving at the speed of light.** *How can you be moving if you are at rest in a chair? You are moving through time.*" [15-78f]

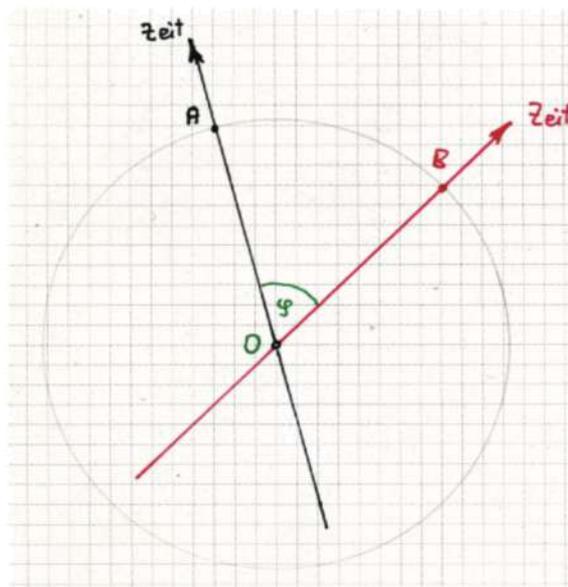
Thus the whole phenomena of the STR result from the fact that we are not all moving in the same direction of four-dimensional space-time! We want to show this graphically and we will see that Epstein diagrams (as they must be called) not only show the phenomena qualitatively correctly, but that all three basic phenomena can be quantitatively correctly read from these diagrams. For many problems one needs only a sheet of graph paper, a compass and a ruler; one draws the suitable Epstein diagram and immediately the solution can be read to two or three decimal places!

## C2 Epstein Diagrams

The dogma reads: Everybody is always and everywhere moving at the speed of light  $c$  in 4D space-time. Everyone calls the direction in which they move their time and the directions orthogonal to it form their space. One second of time corresponds to 299,792,458 m ( $\approx 300,000$  km) of space (that one did not notice this earlier can be explained as a consequence of this 'disproportionateness'...).

If we want to plot this movement in 4D space-time then we have the same problem as everyone else who attempts to draw a four-dimensional representation. One must be happy when a three-dimensional picture is clearly understood on a flat sheet of paper. In our case however these difficulties are easily eliminated: We represent only *one* direction in space, the one in which the two reference systems move relative to each other! Anyway, nothing exciting happens in the other two spatial directions according to our formulas of **B3**!

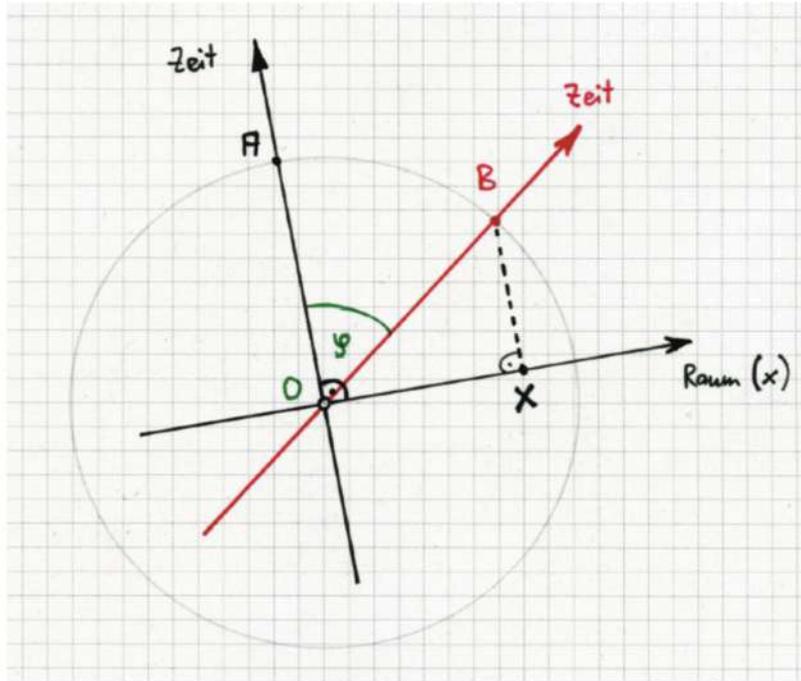
We will always use a black coordinate system with origin A and a red one with origin B, like the one shown above in **B3**. B moves with velocity  $v$  along the x-axis of black and A moves with velocity  $-v$  along the x'-axis of red. A and B met at O and at that point both set their clocks to zero. In addition, everyone tells time with the clocks of their respective system which have been synchronized respective to the master clocks of their system. A and B are each spatially at rest in their own system and move (in their own system) only through time. Our first attempt:



In order to avoid complications with causality we must forbid that red, which had an interaction at O with A, can ever influence the temporal past of black before point O. This means that the angle  $\phi$  may not be larger than  $90^\circ$ . Otherwise red would be able to ignite its engine after some time, return to point O and arrive there at a time before the interaction between A and B, which already took place. We adhere emphatically to the following:

**For systems, which can interact with one another, the angle  $\phi$  between the two time axes (that is, the directions of the journey through 4D space-time) may not be larger than  $90^\circ$ .**

It is important that the segments OA and OB are equal: In a given interval of time both cover the same distance in space-time! That is Epstein's dogma. What is the meaning of the angle between the time axes? We depict the direction of the x-axis of black and mark the place, which B has reached in this x-direction, while black has just aged by the segment OA:



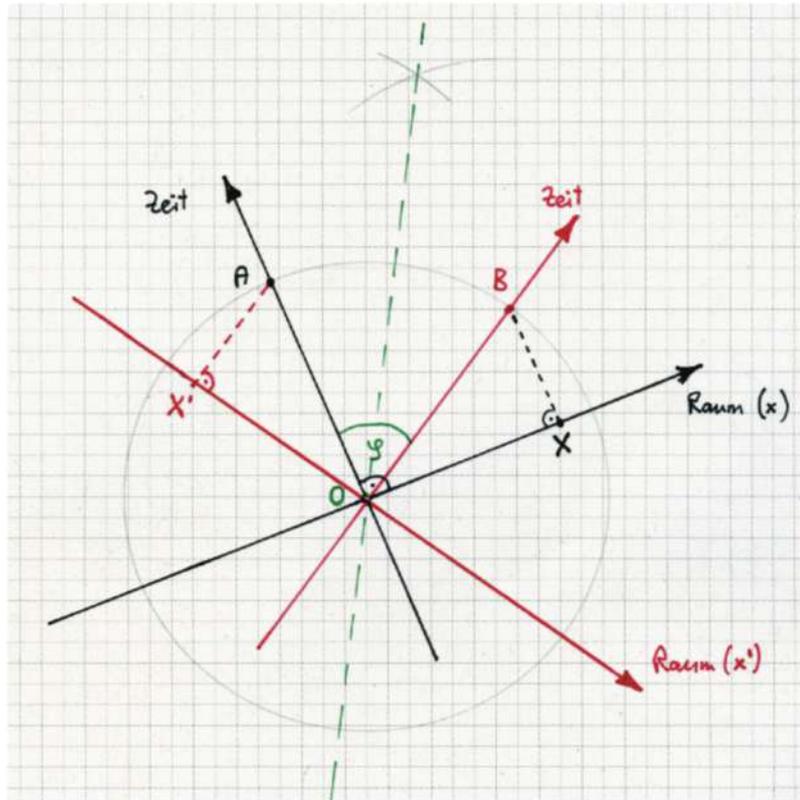
Thus  $OA = OB$ . OA is for black simply the time, which elapsed since the meeting with B at O. B has in this time, from the point of view of black, traversed the segment OX. This yields

$$v = \frac{OX}{OA} = \frac{OX}{OB} = \sin(\varphi)$$

since in the right triangle OXB angle  $\varphi$  occurs again at vertex B. We obtain B's position in the coordinate system of A simply by projecting the space-time position of B perpendicularly onto the space-axis of A. In so doing we are being rather cavalier with our mix of units: OA, OB and OX are segments in space-time. When reducing to pure times and distances we must consider that 1 second of time corresponds to a distance of 1 light-second, or about 300,000 km! If we clean up the units of measurement in the above equation it looks as follows:

$$\frac{v}{c} = \frac{v \cdot \Delta t}{c \cdot \Delta t} = \frac{OX}{OA} = \frac{OX}{OB} = \sin(\varphi)$$

The unit-free number  $\sin(\varphi)$  corresponds in the Epstein diagram to the ratio  $v$  to  $c$ ! So far we have taken the point of view of black. That is unnecessary since Epstein diagrams have (unlike Minkowski diagrams!) the beautiful characteristic that they display symmetrical relationships symmetrically. Thus, we draw the above diagram again with the addition of a space-axis for B:



From the point of view of A: During the time OA, B traverses OX.  
 From the point of view of B: During the time OB, A traverses OX'.

The two triangles OXB and OX'A are congruent. The same absolute value for the relative velocity  $v$  arises in both coordinate systems. Because of the selected orientation of the axes we obtain however different signs for  $v$ : For red A moves in negative  $x'$ -direction, while for black B moves in positive  $x$ -direction. Thus we are in complete agreement with the presentation in **B3**.

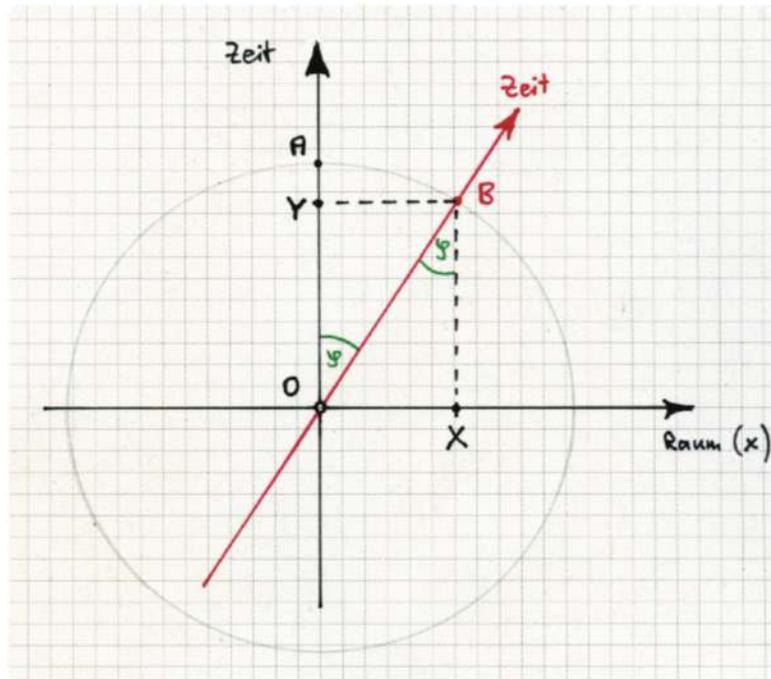
Perhaps you are surprised that the time axes are not indicated with  $t$  or  $t'$  (Note: in the diagrams the German word 'Zeit' stands for 'time', where as 'Raum' stands for 'space'). We reveal the reason for this in the next section. First we want to do another small calculation, which yields a very important result. For acute angles  $\phi$  we have

$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \left(\frac{v}{c}\right)^2} = \sqrt{1 - \sin^2(\phi)} = \sqrt{\cos^2(\phi)} = \cos(\phi)$$

The radical which appears in the calculation of time dilation and length contraction, has a simple geometrical meaning in the Epstein diagram! Spoken as black:  $\sin(\phi)$  projects OB on my space-axis,  $\cos(\phi)$  projects OB on my time-axis. We would like to exploit that immediately.

### C3 Time Dilation in the Epstein Diagram

Thus, for black, B moves with velocity  $v$  in the direction of the positive  $x$ -axis. With the meeting at  $O$  both A and B set their clocks to 0000. The meeting took place at  $x = 0$  and  $x' = 0$ . Then A and B moved on in space-time (with the speed of light) and somewhat later we have the following situation:



According to **C2**,  $OA = OB = c \cdot \Delta t$  and B has moved spatially for black from  $O$  to  $X$ . Thus  $OX = OB \cdot \sin(\varphi) = (c \cdot \Delta t) \cdot (v/c) = v \cdot \Delta t$ .

What is the meaning of the segment  $OY$ ?  $OY = OB \cdot \cos(\varphi) = OA \cdot \cos(\varphi)$

If we no longer interpret  $OA$  in space-time, but rather assume the point of view of black and consider  $OA$  as "the time  $\Delta t$  that has passed for me since our meeting at  $O$ ", then  $OY$  represents  $\Delta t \cdot \cos(\varphi)$ , and that is nothing else but  $\Delta t'$  (according to the statement at the end of **C2**). That is, it is the time, from the point of view of black (and also from that of red!) that has elapsed for red since the meeting in  $O$ ! Since  $\cos(\varphi)$  corresponds directly to the radical term which we use in the time dilatation calculation, black can directly read from its time-axis, how much time elapsed *for everyone* since the meeting in  $O$ .

To repeat: Black projects the space-time position of everyone on its time-axis and therefore knows how much time elapsed for everyone since the meeting in  $O$ . Since the clocks were set to zero at the meeting, we have

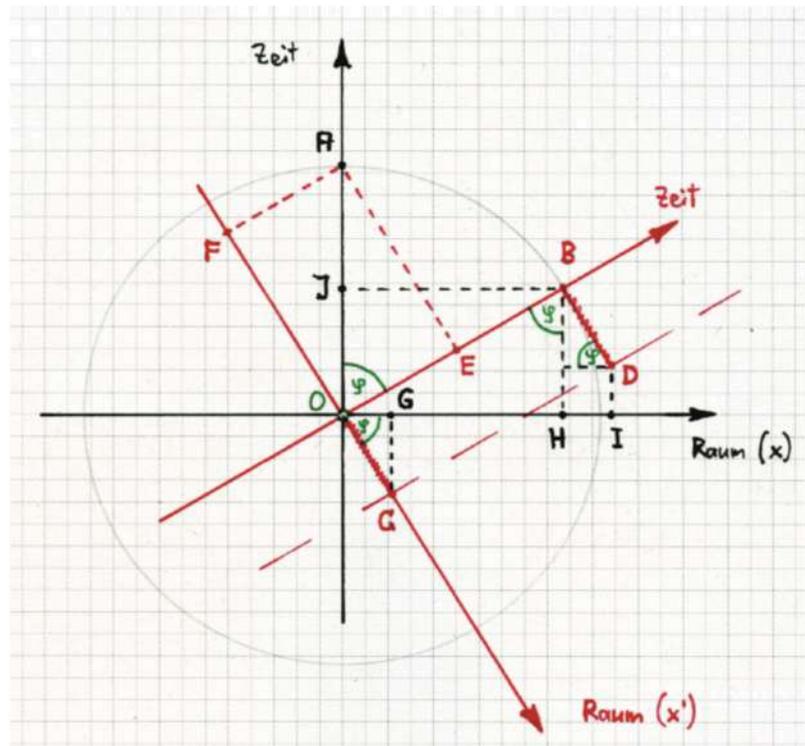
$$OA = \Delta t = t \quad \text{and} \quad OY = \Delta t' = t'$$

**From its time-axis black reads  $t$  and  $t'$ , obtaining the elapsed time of all clocks involved!**

We can avoid the conversion using factor  $c$ , if we compare only times. The whole representation would be simpler and more elegant, if we would measure lengths in light seconds.  $c$  would then simply have the value 1 or 1 light-second per second, that is '1 light'. However there are good reasons to stick to the standard system of units.

## C4 Length Contraction in the Epstein Diagram

Also the length contraction can be easily and quantitatively correctly read from the Epstein diagram. Consider the following diagram:



What does red say?

The segment OC is at rest in my system and it is only getting older. It moves through time by  $OB = CD$ . The segment has the length  $OC = BD$  which is its proper length. Black moves in space-time an equivalent distance, i.e. from O to A. Purely spatially speaking it moves with  $v = -c \cdot \sin(\phi)$  from O to F.

Therefore less time elapses for black, only  $OE = OA \cdot \cos(\phi) = OB \cdot \cos(\phi)$

What does black say?

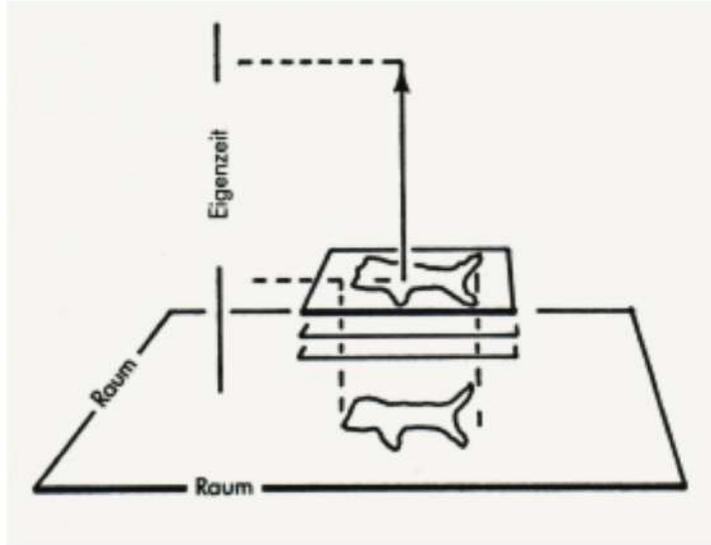
The segment OC moves in space-time toward BD while I simply grow older by OA. Meanwhile the segment OC ages only by  $OJ = OB \cdot \cos(\phi) = OA \cdot \cos(\phi)$ . I measure a length of  $OG = HI$  in the x-direction for this segment. It is  $OG = OC \cdot \cos(\phi)$ . Purely spatially I measure that a segment OG moves toward HI with speed  $v = c \cdot \sin(\phi)$ .

All statements are in complete agreement with the results of **B2** and **B3**!

In its own system each object has its proper length and moves only through time. The tilt of angle  $\phi$  causes the lengths of fast objects to be measured as shortened 'shadows'. In addition, the progression of the time is slowed for these fast objects. The beauty of this is that both effects are quantitatively correctly shown. Also: The two principles of the maximum proper time and the maximum proper length become evident in the Epstein diagram!

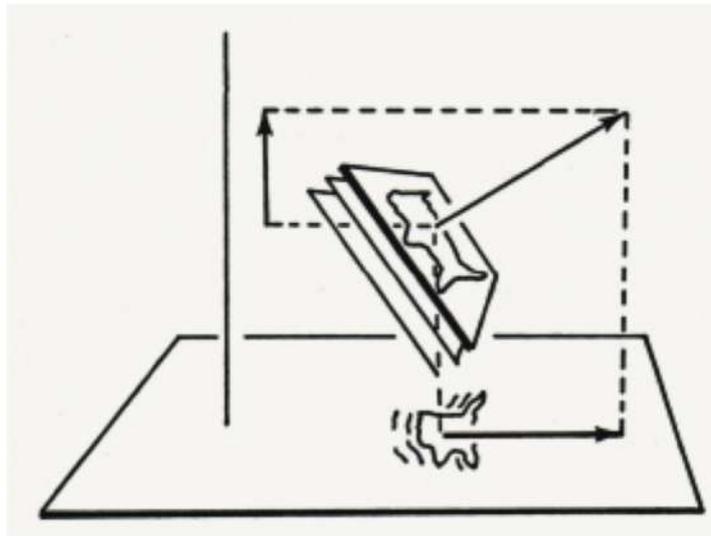
From length contraction, as described in **B3**, only the direction of movement in space is affected, which agrees with the direction of the relative velocity. Epstein illustrates this with two diagrams [15-91]:

1. The USA are at rest. It moves only through time:



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2. The USA are observed from above by a spacecraft racing from east to west. For passengers in the spacecraft the USA is therefore racing from west to east. The time axis of the USA tilts through an angle  $\phi$  and the spatial projection of the USA shrinks, but only in the direction of the relative motion:

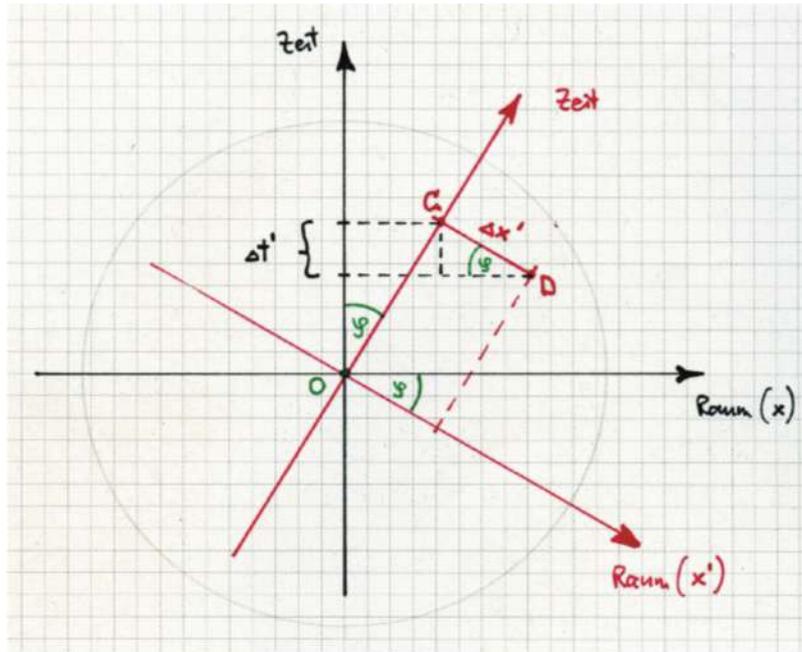


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Space and time actually shrink down to mere shadows (see Minkowski quote of **C1**), shadows of the movement through space-time, trapped in the speed of light!

## C5 Desynchronization in the Epstein Diagram

Clocks, which are synchronized for red, project onto the same point of the red time-axis. All red synchronized clocks lie on lines parallel to the red space-axis. For black, however, which reads time always from its black time-axis, such clocks exhibit a time difference:



Black ascertains:  $\Delta t' \cdot c = \Delta x' \cdot \sin(\varphi) = \Delta x' \cdot v/c$ , which is, except for the sign, precisely the formula of **B6**! In addition, we can determine the sign: From the point of view of black, the *leading* clock of red at D runs *behind* the spatially trailing clock at C. Therefore it must mean

$$\Delta t' \cdot c = -\Delta x' \cdot \sin(\varphi) = -\Delta x' \cdot v/c$$

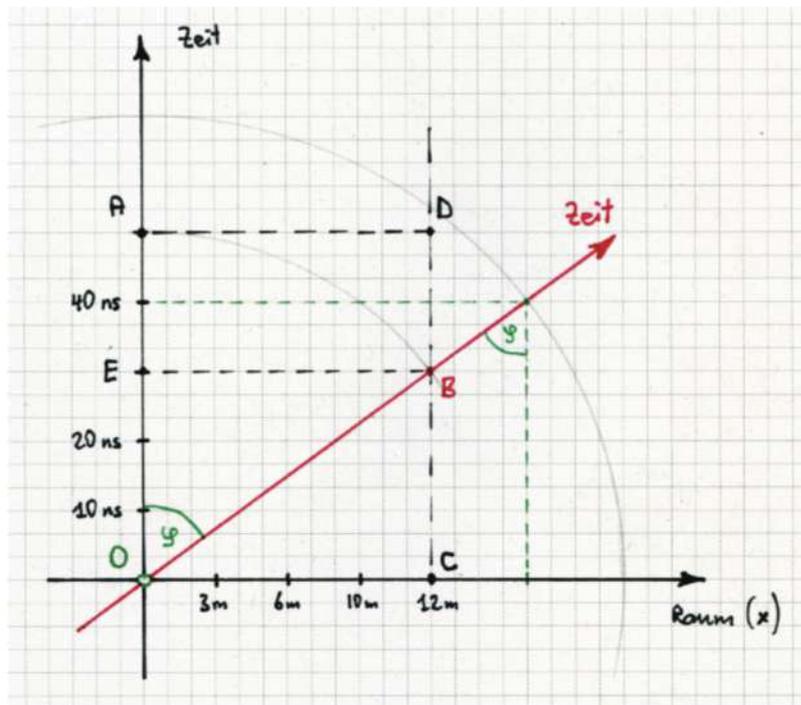
in perfect agreement with **B6**. Both red clocks are moving at the same speed, but the one in front (spatially) is permanently behind (temporally) the other by the same amount.

It is worth considering (and also non-trivial) how the synchronization of distant passing clocks of **B6** would look in an Epstein diagram. One must note the fact that light in each reference frame always moves with  $c$  and thus the quotient  $v/c$  is 1 and the angle  $\varphi$  is always  $90^\circ$ ! Light moves in each reference frame only spatially, parallel to the space-axis. Thus: Photons do not age!

## C6 Our Sample Problem as Epstein Diagram

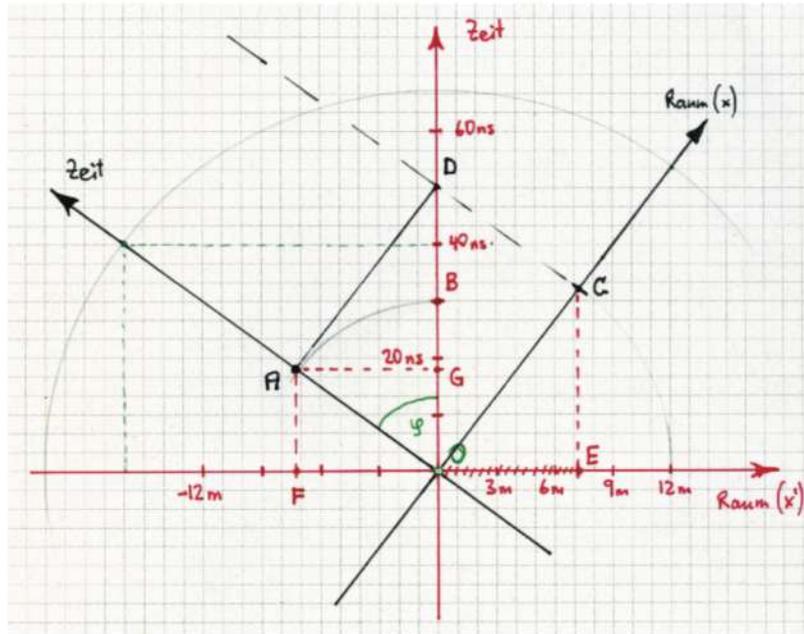
We want to present the sample problem, which we solved at the end of **B6** as an Epstein diagram. To keep everything as simple and clear as possible we will draw two diagrams: One from the point of view of black with a pipe at rest and the other from 'the red' point of view of the particle, which sees the pipe racing by.

First to the angle between the two time-axes: The sine value of  $v/c = 0.8$  need not be converted into an angle. We need only count the little squares on our graph paper and select 20 squares for the circle radius, e.g.:  $16/20 = 8/10$  provides the correct angle (note the green auxiliary lines).



While the particle moves in space-time from O to B, the pipe ages according to the same segment, that is,  $OA = CD$ . The clocks at the two tube ends are synchronized for black: O and C (later A and D) project on the same point of the black time-axis. Reaching point B the particle leaves the pipe. The units are selected as follows: 3 squares correspond spatially to a distance of 3 m and to an interval of 10 ns on the time-axis! Importantly: One is free to select only one of the two scales, the other then being fixed by the speed of light. We read: The transit lasts for black  $OB = OA = 50$  ns, while only  $OE = 30$  ns applies for red.

That is the view of black. Now we consider the point of view of red.



For red the pipe OC moves through space-time to AD. This requires a time of  $OA = CD = OB = 30$  ns. Red measures the length of the pipe to be  $OE = FO = OC \cdot \cos(\phi) = OC \cdot 0.6 = 7.2$  m. Red knows that the clock of black indicates 50 ns at point D. Red says nevertheless that the whole transit required the time  $OG = FA = OA \cdot \cos(\phi) = OB \cdot \cos(\phi) = 30 \text{ ns} \cdot 0.6 = 18$  ns for black. Black measures 50 ns, because its rear clock (the one that moves on segment CD) exhibits a constant advance of  $EC = GD = 32$  ns over the other one moving on OA. This is the desynchronization of black's clocks as stated by red.

Carefully compare this presentation with **B6** and convince yourself that we can read all results correctly from the diagrams without the need to do any calculations!

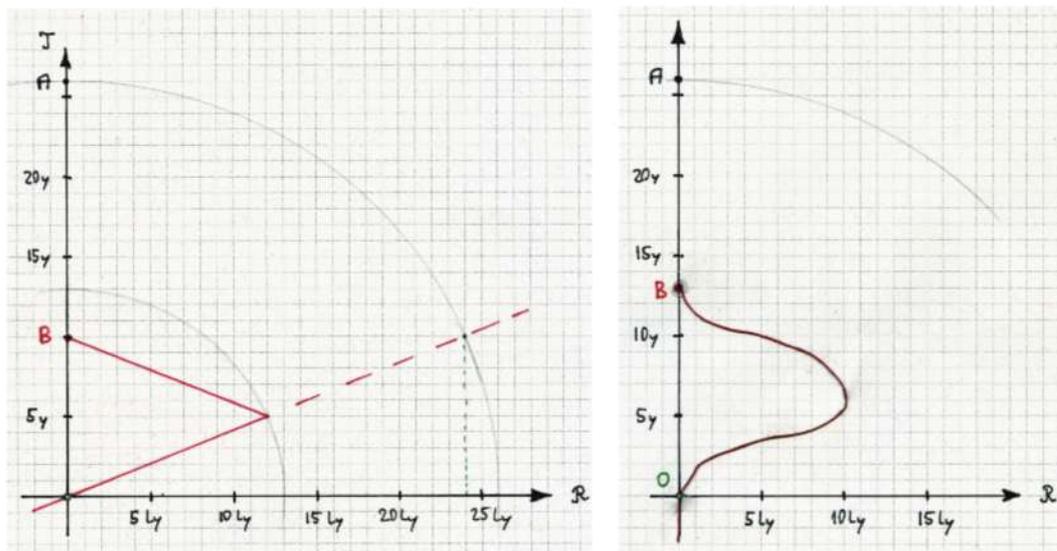
You can easily practice using Epstein diagrams: download the program appropriate for your operating system from [www.relativity.li/en/diagrams/application/](http://www.relativity.li/en/diagrams/application/) !

## C7 Twin Paradox as Seen By Epstein

The following story has disconcerted thinkers time and again: Twins A and B both study space travel. At age 25 B sets off on a prolonged space journey with  $v = 12/13 \cdot c$ , while A serves in the control center on earth. After 26 years earth time A is precisely 51 years old, as his brother B returns to earth from his journey. How old is B?

We want to ignore the short acceleration phases when starting, reversing flight direction and landing and assume that B flew away for 13 years with velocity  $v$  and afterwards flew back 13 years with velocity  $-v$ . The radical term,  $\cos(\phi)$ , is  $5/13$  for our value of  $v/c$ . Therefore, for each of the out-bound flight and the return flight B aged only 5 years. On his return to earth he is therefore only 35 years old, while his twin brother is celebrating his fifty-first.

“From the point of view of the space traveler is everything reversed, and thus A must be younger than B! Thus the whistle is blown on this swindle of relativity theory.” Heated lampoons against the STR are expressed thus or similarly. Actually, however, the whole arrangement is asymmetrical: Only A is the whole time at rest in an inertial frame, while B is exposed in different phases of his journey to accelerations. During the non-accelerated flight phases B actually has the impression that his brother A works somewhat slowly. The Epstein diagram can present the whole situation beautifully:

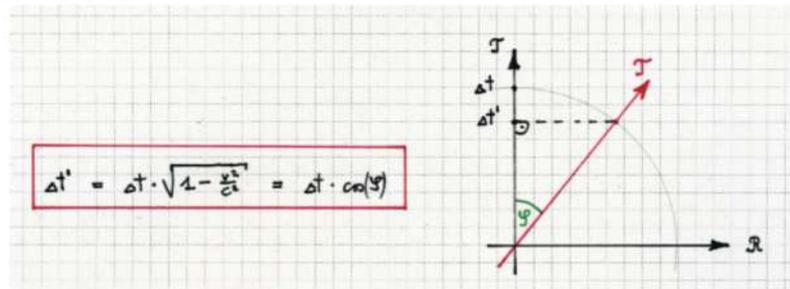


The diagram on the left corresponds to the text while the one on the right is somewhat more realistic, since it considers the acceleration phases. It is important that the paths through space-time for both twins are equivalently long: If one straightens the red thread, then one has a length of exactly 26 squares from O to B, and is thus the same length as OA. Since B claims a part of the space-time segment for the spatial distance there remains less for his movement in time. It really is so simple!

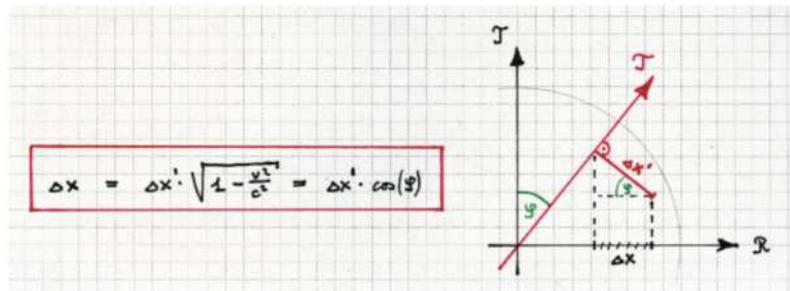
Compare this representation of the twin paradox with others which use Minkowski diagrams, e.g. in [18-64] or [19-146f] !

## C8 Summary

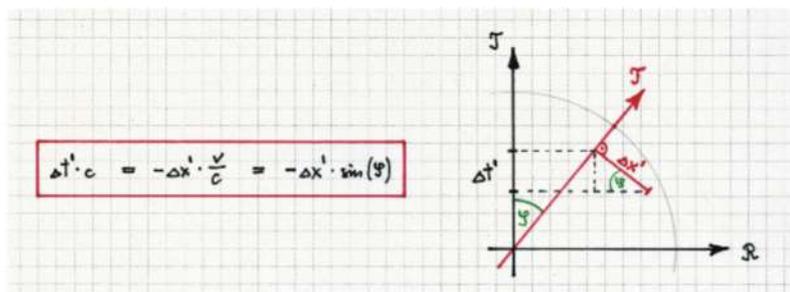
1. Time dilation - Relativity of time measurement - "fast clocks run slow"



2. Length contraction - Relativity of length measurement - "fast yardsticks are shorter"



3. Relativity of simultaneousness - the synchronization of clocks at rest is valid only within their own inertial frame



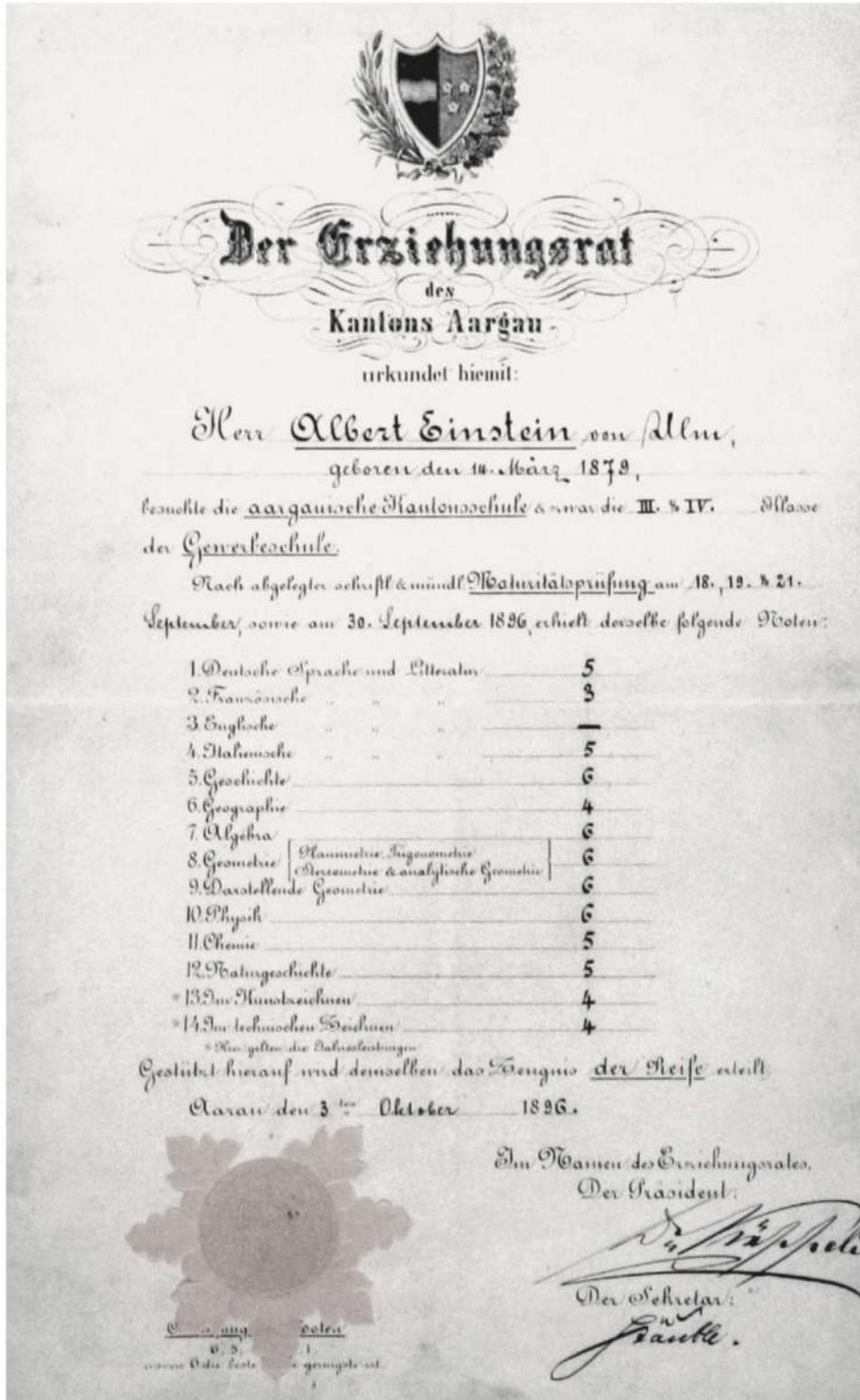
4. The formulas:

$$\sin(\varphi) = \frac{v}{c}$$

$$\cos(\varphi) = \sqrt{1 - \frac{v^2}{c^2}}$$

## C9 Problems and Suggestions

1. Draw an Epstein diagram of an airplane B thundering over the head of A at 3240 km/h.
2. Solve problems 6, 7 and 11 from **B7** using Epstein diagrams.
3. How fast does an Einstein train of 260 m proper length have to travel, so that it fits completely into a tunnel of proper length 240 m? In which system are we thinking, when we formulate the question in this way?
4. Positive pions are so fast that their radioactive half-life is quadruple in comparison to the value one measures of slow pions. Determine the speed of these pions a) computationally and b) graphically.
5. The famous problem of the 6 m long car and the 6 m long garage, which is provided with a door in the front and in the back: The car races with  $0.8 \cdot c$  through the garage. At the beginning the front door is opened and the back closed. As soon as the car is completely in the garage the front door closes. How long is the car completely in the garage? Which length does it have in the system of the garage? When, at the latest, does one have to open the back door?
6. Look again at problem 5, but this time from the point of view of the car, over which the strange garage tube is racing. How long is the tube? Why do the people in the system of the garage think they could for a certain time completely lock the car up?
7. 4 space stations, each at rest relative to the others, are to form a large square whose sides have length 1 light second. An observer X flies exactly along the diagonal AC with velocity  $0.8 \cdot c$ .
  - a) What shape does the square have for X? (calculation and picture)
  - b) How long does this flight from A to C last from the view of the square inhabitants?
  - c) The square inhabitants know that for the duration of the flight X measures another value. What is the value, and what is the reason for it from the view of the square inhabitants?
  - d) X actually measures the same value, however his justification for it is completely different. How?
  - e) The inhabitants of the square synchronized their clocks in A, B, C and D. For X however only 2 of these clocks are synchronized. Which?
  - f) Which desynchronization do the clocks of A and B exhibit for X?
8. Since the Big Bang (see cartoon in **I2** !) about 14 billion years have past. How old is a galaxy today for us, which since the Big Bang has been moving away from our Milky Way with  $0.9 \cdot c$ ? And when was the light sent by this galaxy, if it reaches us today? Draw an Epstein diagram!
9. B flies with his space scooter in a straight line past the earth A with  $0.8 \cdot c$ . On his high precision wrist-watch B ascertains 30 minutes after the earth flyby that he passes a space station C. At this point he sends a radiogram to the earth. C and A are at rest relative to each other and their clocks are synchronized.
  - a) What distance do A and C have for B?
  - b) What duration do the people in A and C measure for the flight of B over this distance?
  - c) How many minutes after the flyby of B at A does the radiogram of B arrive at A?Try using an Epstein diagram!



Einstein's 'entry ticket' to the Swiss Federal Institute of Technology (ETH Zürich): The Matura diploma of the Kantonsschule Aarau. It is often repeated that he was a weak student, but this is a misinterpretation: In Germany, 1 is the best mark and 6 the worst, while in Switzerland the scale runs from 6 for the best mark down to 1.



Spiral Galaxy NGC 3190  
(FORS/VLT)

ESO Press Photo 17/06 (11 May 2006)

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## D Lorentz Transformations, Velocity Addition and Doppler Effect

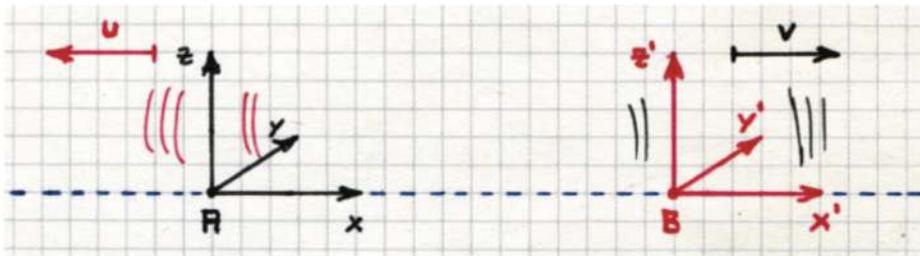
We examine the transformation of coordinates between different inertial frames in the mechanics of Galileo and Newton and deduce the old formula for the addition of velocities. Then we do the same considering the 3 fundamental effects of STR. We deduce the so-called Lorentz transformations twice: First with assistance of Epstein diagrams and a second time only using the formulas from section **B**. From the Lorentz transformations we obtain the relativistic formula for the addition of velocities. Finally we deduce the optical Doppler formula.

## D1 Coordinate Transformations before the STR

A physical *event* takes place at a certain time and at a certain place. Thus, in each inertial frame (coordinate system, reference frame) we can assign to an event a time coordinate as well as three local coordinates. In each coordinate system one point of 4D space-time belongs to an event.

We examine in this and the following two sections how the 4 coordinates assigned by an observer to an event in one inertial frame are correctly converted to the corresponding 4 coordinates assigned by a second observer of the same event in another inertial frame. The formulas describing this conversion are called *coordinate transformations*.

We assume (as previously) two coordinate systems - a black, non-prime system and a red, prime system, which are aligned to each other in a simple manner (same diagram as in **B3**):



The origin B of the red system moves with velocity  $v$  along the  $x$ -axis of black, and the  $x'$ -axis of red coincides with the  $x$ -axis of black. Thus A moves with velocity  $u = -v$  along the  $x'$ -axis of red. The two other spatial axes ( $y/y'$  and  $z/z'$ ) will always be parallel to each other. In addition, both red and black set their clocks to zero at the moment when they coincided. All other clocks that black possibly uses are synchronized within its frame with the master clock in A. And all 'red' clocks are synchronized with the red clock B within the red system.

We now consider coordinate conversions within the mechanics of Galileo and Newton: Since the clocks were set to zero by red and black at their meeting, both clocks (as well as all other clocks of red and black) will always agree. They agree to the degree of their accuracy, velocity of movement has no influence on the synchronization or on the clock tick rate. Thus for each moment and in all places it applies

$$(1) \quad t = t'$$

Also concerning the distances from the  $x$ -axis, which is identical to the  $x'$ -axis, one will find no differences. That is

$$(2) \quad y = y' \quad \text{and} \quad z = z'$$

There is something to convert only if one wants to convert an  $x'$ -coordinate of red to  $x$  or vice-versa.  $x'$  is the distance from the point of the event, projected on to the  $x/x'$  line, to the red origin B. Origin B has the distance  $v \cdot t$  from the black origin A. Thus the  $x$ -coordinate of the event is calculated simply as

$$(3) \quad x = x' + v \cdot t = x' + v \cdot t' \quad \text{and correspondingly} \quad x' = x - v \cdot t$$

We have just derived the very simple Galileo transformations. We have crucially made use of Newton's concept of absolute time, which is the same for everyone, as well as his concept of absolute space, which permits the calculation of absolutely valid distances or lengths.

We will now demonstrate (as promised in **A3**) that the 'classical' addition of velocity results from this basis of Galileo and Newton ('classic' is always equivalent to 'not relativistic' in this context).

Assume C moves in the red system with the speed of  $w'$  in the  $x'$ -direction. The  $x'$ -coordinate of C is thus  $x' = a + w' \cdot t'$ , where  $a$  is any constant. This is the distance in the  $x'$ -direction from B to C. B is however at any given time  $t$ , as seen from black, at the point  $v \cdot t$ . The point of C along the  $x$ -axis of black is thereby  $x = v \cdot t + a + w' \cdot t'$ . We thus have already used the absoluteness of space. Now we use Newton's absolute time and simply replace  $t'$  by  $t$  using (1). Thus we obtain

$$x = v \cdot t + a + w' \cdot t = a + (v + w') \cdot t.$$

So the  $x$ -position of C increases with speed  $v + w'$  for black. Speeds simply add in classical mechanics.

In order to compare with the somewhat more complicated Lorentz transformations, which we will deduce in the next section, we present the Galileo transformations in the following table:

$$\begin{aligned} t' &= t \\ x' &= x - v \cdot t \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} t &= t' \\ x &= x' + v \cdot t' \\ y &= y' \\ z &= z' \end{aligned}$$

These coordinate transformations describe how in classical mechanics the coordinates  $(t, x, y, z)$ , which black attributes to an event should be converted into the coordinates  $(t', x', y', z')$ , which red attributes to the same event - and vice-versa. If the two coordinate systems were less precisely aligned to each other, then naturally the lines 2, 3 and 4 in the small boxes would be somewhat more complicated. Nothing at all would change however in the first line, which reflects Newton's absolute time.

A somewhat childish suggestion: Go against the official direction of motion on an escalator (as you surely once did as a child) or on one of the long "moving sidewalks" one finds in airports. It is fun and also a direct way to experience the addition (or rather subtraction) of velocities.

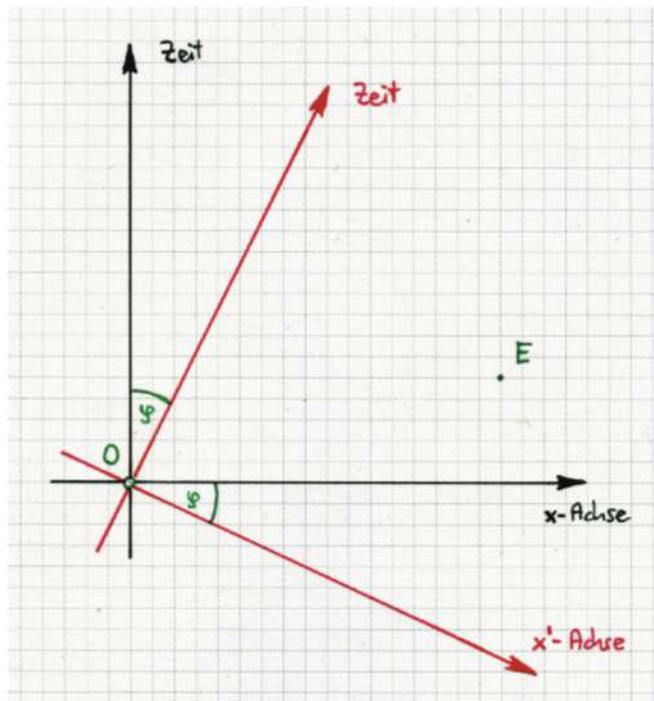
## D2 Derivation of the Lorentz Transformations from Epstein Diagrams

If black and red want to compare their measured values for place and time which they assign to an event, then it presupposes that they have already crossed paths and synchronized their clocks. Synchronization means that both set their "master clocks" at this meeting at  $x = 0 = x'$  to  $t = 0 = t'$  and synchronized any other clocks within their system with the master clock. To compare the coordinates of an event is always to talk about a *second* contact of red and black. Otherwise the measured values of red and black would be arbitrary! This remark by the way applies to all space-time diagrams, not only to those of Epstein.

Consider two coordinate systems meeting at O as described in the preceding section. Now a red clock moves by a particular location of black, and the coordinates  $(t', x', y', z')$  are recorded. Which coordinates  $(t, x, y, z)$  will black attribute to this meeting? How can we, in general, convert such event coordinates, given that we account for the relativistic effects of time dilation, length contraction and desynchronization derived in section B from Einstein's basic postulates?

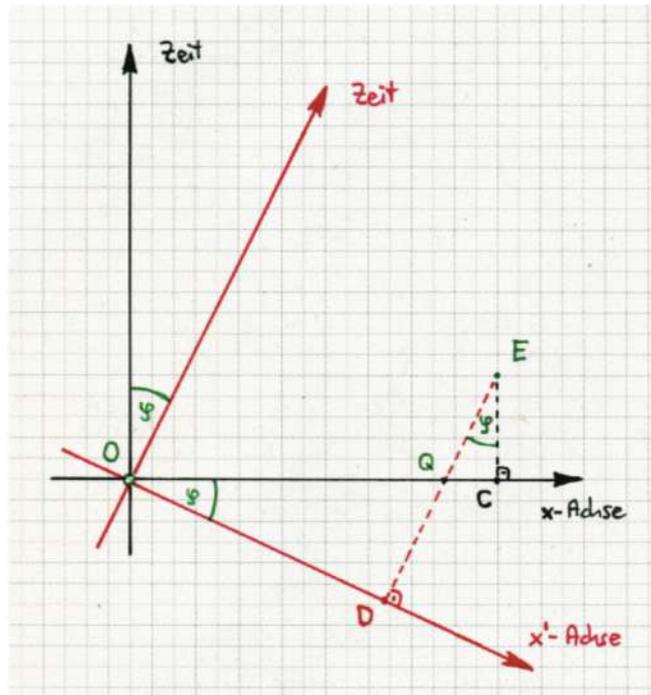
This question is answered by the Lorentz transformations already mentioned in A3 and A4. In this section we derive the Lorentz transformations from Epstein diagrams. For skeptics a second derivation follows in the next section based only on the three basic phenomena and their quantitative description in section B.

First we note that the convenient equations  $y = y'$  and  $z = z'$  still apply. As described in B3 distances perpendicular to the relative velocity of the two systems, that is, perpendicular to  $x$  and  $x'$ , are the same for both black and red. We need only worry about the time coordinates and the spatial coordinates in the direction of relative motion  $v$ . And it is exactly these values which are perfectly represented in the Epstein diagram. We start with a nearly 'empty' Epstein diagram:

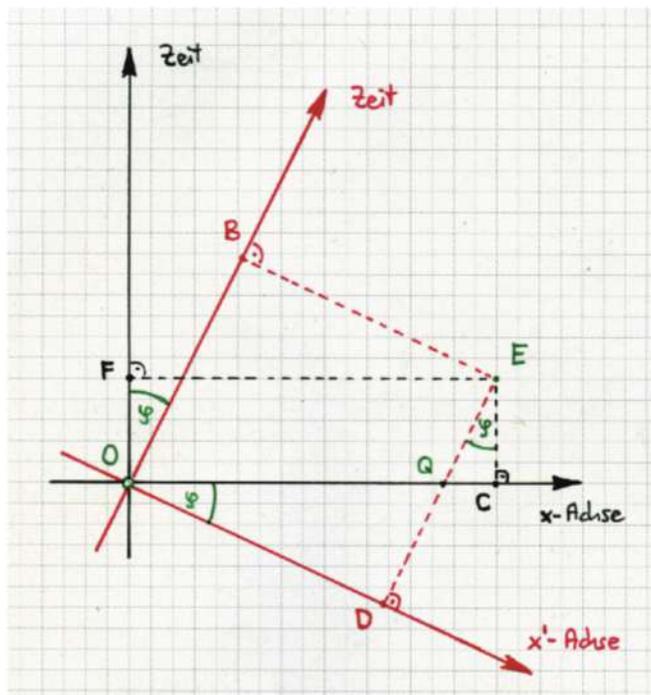


We mark an arbitrary point E in space-time. No doubts exist about where E lies in black and red: We need only project E onto the  $x$ -axis, respectively  $x'$ -axis. We obtain the points C and D (in the following figure). These projections additionally yield the auxiliary point Q, which we will later make use of. We have

(1)  $x = OC$  and  $x' = OD$  and still (2)  $y = y'$  and  $z = z'$



E defines the event "the red clock of  $x'$  is at location  $x$  of black". The time  $t'$  of this event can only be measured with the red clock which is locally present! Now we draw the projections on the two time axes and obtain the points F and B:

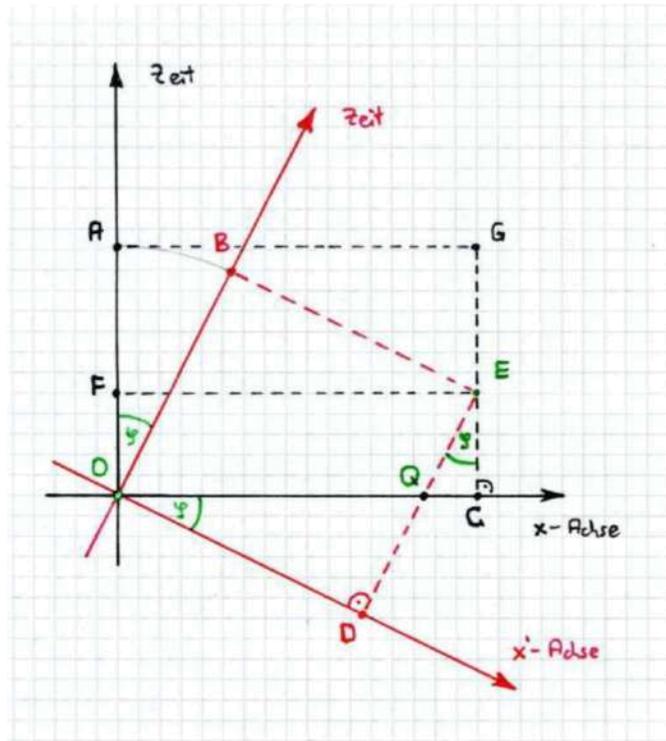


For red the clocks at O and D (or, somewhat later, at B and E) are synchronized. Black sees things differently. The time indicated by the local red clock for event E, is for black simply

(3)  $t' = OF$

Thus we have accounted for the two effects of time dilation and desynchronization!

The time  $t$  of black is still missing in the diagram. Which clock does black use to measure the flyby of the red clock at location  $x$ ? Clearly it uses the one at location  $x$ , that is, the one which was at point  $C$  when the meeting of the master clocks took place at point  $O$ . However, where is this clock, given that the red clock has moved from  $D$  to  $E$ ? According to the dogma of Epstein everything moves equally far through space-time between two events. The black clock at location  $x$  is therefore at point  $G$ , and therefore we have by necessity  $CG = OA = OB = DE$ .



The two points  $E$  and  $G$  in the Epstein diagram *both* belong to the event “the red clock in its system at location  $x'$ , flies past location  $x$  of black” ! This is often confusing for those, who are well versed with space-time diagrams for which an event always corresponds to a *single* point in the diagram.

So black attributes the following time coordinate to this event:

$$(4) \quad t = OA = OB = DE$$

We still have to find the values  $(t, x, y, z)$  for the associated values  $(t', x', y', z')$  and vice-versa. We already know how it is with  $y$  and  $z$ . We now investigate how to obtain the values  $(t, x)$  from  $(t', x')$ . The reverse transformations will be left as a small algebra exercise. We use lengths for all space-time distances and must therefore multiply time values by the speed of light  $c$ . Thus:

$$t \cdot c = OA = OB = DE = DQ + QE = OD \cdot \tan(\phi) + CE / \cos(\phi) = x' \cdot \sin(\phi) / \cos(\phi) + t' \cdot c / \cos(\phi)$$

Recalling the meaning of  $\sin(\phi)$  and  $\cos(\phi)$ , we are already finished:

$$t \cdot c = x' \cdot (v / c) / \sqrt{1 - v^2 / c^2} + t' \cdot c / \sqrt{1 - v^2 / c^2} = (t' \cdot c + x' \cdot v / c) / \sqrt{1 - v^2 / c^2}$$

Dividing by  $c$  and writing the whole somewhat more conventionally, yields

$$t = \frac{t' + \frac{x'}{c} \cdot \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The two terms in the numerator beautifully represent (together with the denominator) the effects of time dilation and desynchronization. For  $x' = 0$  we have only the first effect whereas  $t' = 0$  underscores the second effect.

Just as easily we can derive how one obtains  $x$  from  $(t', x')$ :

$$x = OC = OQ + QC = OD / \cos(\phi) + EC \cdot \tan(\phi) = x' / \cos(\phi) + t' \cdot c \cdot \sin(\phi) / \cos(\phi) \quad \text{thus}$$

$$x = x' / \sqrt{1 - v^2/c^2} + (t' \cdot c \cdot v / c) / \sqrt{1 - v^2/c^2} = (x' + v \cdot t') / \sqrt{1 - v^2/c^2}$$

Here the difference to the corresponding Galileo transformation is less significant, showing as it were only length contraction.

Here are the resulting transformations:

$$\begin{aligned} t' &= \frac{t - \frac{x}{c} \cdot \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \\ x' &= \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} t &= \frac{t' + \frac{x'}{c} \cdot \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \\ x &= \frac{x' + v \cdot t'}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y &= y' \\ z &= z' \end{aligned}$$

The transformations  $(t, x, y, z) \leftrightarrow (t', x', y', z')$  presented here must mutually cancel, if they are executed consecutively. This calculation is recommended to the reader as an exercise.

As author of this book I would like to sing the highest praise for the Epstein diagram. In the USA one would call me an 'evangelist'. I am indeed quite proud to be the first Epstein evangelist to show the derivation of the Lorentz transformations from an Epstein diagram ...

For the skeptics or otherwise incorrigible non-believers a derivation of the transformations follows in the next section which is completely devoid of Epstein diagrams and based only on the results of section **B**.

### D3 Derivation of the Lorentz Transformations from Basic Phenomena

Consider again two coordinate systems, as described at the beginning of **D1**. An event E takes place for red at time  $t'$  at the point  $(x', y', z')$ . We now know that it takes place for black at the point  $(x, y, z)$  with  $y = y'$  and  $z = z'$ . Still black's values for  $t$  and  $x$  are to be determined for this event.

For black the 'master clock' of red at location B (like each clock of red) runs too slowly, thus

$$t = t(A) = t'(B)/\sqrt{\quad} \quad \text{or} \quad t \cdot \sqrt{\quad} = t'(B)$$

A clock at location  $x'$  of red, in addition, shows for black a desynchronization to the clock at B of

$$\Delta t' = -x' \cdot v/c^2$$

Thus the red clock at B already shows  $t'(B) = t'(x') - \Delta t' = t' + x' \cdot v/c^2$ , when E takes place. Thus we obtain the corresponding clock state for A from the expression

$$t(A) \cdot \sqrt{\quad} = t'(B) = t' + x' \cdot v/c^2$$

For  $t = t(A)$  itself we obtain  $t = (t' + x' \cdot v/c^2) / \sqrt{\quad}$

It was precisely this expression for  $t$  that we derived in **D2**!

We still must clarify, at which  $x$ -coordinate the event E takes place for black. Red thinks that the distance  $d'$  from A to the location  $x'$  of the  $x'$ -coordinate of E has the following value:

$$d' = x'(E) - x'(A) = x' + v \cdot t'(B) = x' + v \cdot t'(x') = x' + v \cdot t'$$

Here we have used the fact that for red the clocks at B and at  $x'$  are synchronized. Thus for red the event E has the distance  $d'$  from A along the  $x$ -axis, as well as along the  $x'$ -axis. For black all measurements of red in the  $x$ -direction are Lorentz contracted. The distance of the event from A must therefore be for black  $d = d'/\sqrt{\quad}$ , and we are finished:

$$x = d = d'/\sqrt{\quad} = (x' + v \cdot t') / \sqrt{\quad}$$

It was precisely this expression for  $x$  that we also derived in **D2**!

It is interesting to note that most high school text books about the STR do not *derive* these Lorentz transformations. Often they are 'assumed' or simply 'stated' and then time dilation and length contraction are derived from them. The reverse path from the basic phenomena to these more abstract transformations can be taken, only after desynchronization has been quantitatively dealt with.

The reason for introducing the Lorentz transformation is however the same with all authors: They provide a simple derivation of the correct formula for the addition of velocities. Here we have to deal in principle with three inertial frames: B moves with  $v$  relative to A, and C moves with  $w'$  relative to B. What speed does C then have for A? The angle  $\phi$  between the time axes can be the same, e.g.  $60^\circ$ , for both A and B and for B and C. If one combines rather naively the two corresponding Epstein diagrams into one with 3 time axes, then the angle between the time axes of A and C is already  $120^\circ$ ! Even I, as Epstein evangelist, was not spared until recently the derivation of the formula for the addition of velocities in the STR via the Lorentz transformations.

Alfred Hepp made me aware of the fact that Epstein shows in the second edition of [15] in appendix A, how one can construct with compass and straight-edge the correct tilting angle for the speed of  $w$  from those for  $v$  and  $w'$ . Thanks to the sketch in that section (which also first requires understanding!) we could finally find a quite simple proof for the addition formula (red box on the following page), which is based only on Epstein diagrams, and with which one can completely avoid Lorentz transformations. I present this proof in the appendix **K7**.

## D4 Addition of Velocities in the STR

In **D1** we showed that the addition of velocities is simple in the mechanics of Galileo and Newton: If B moves with velocity  $v$  in the  $x$ -direction of A and C moves for B with velocity  $w'$  in the same direction, then C moves with velocity  $v + w'$  for A. We recognized in **A3** that this simple formula cannot apply in the STR: The light of a locomotive standing still must travel forward just as fast as that of one moving forward, i.e. with  $c$ .

For the new velocity addition formula it must be true that the sum of  $v$  and  $c$  results in  $c$ . It must also be true that in no case may a velocity be greater than  $c$ . If a spaceship flies past us at  $0.7 \cdot c$  and fires off a rocket in the direction of its flight, which itself has velocity  $0.8 \cdot c$  relative to the spaceship, then we would already have a speed of the rocket of  $1.5 \cdot c$  according to Newton ...

The derivation of the correct formula for the addition of velocities is quite harmless, if the Lorentz transformations are available:

Let C move with velocity  $w'$  in the  $x'$ -direction of B, while B moves as usual with a relative velocity  $v$  along the  $x$ -direction of A. Then for the  $x'$ -coordinate of C we have

$$x' = a + w' \cdot t' \quad \text{where } a \text{ is some constant}$$

We now simply substitute both  $x'$  and  $t'$  by expressions with  $x$  and  $t$  from the Lorentz transformations:

$$x' = (x - v \cdot t) / \sqrt{\quad} \quad \text{and} \quad t' = (t - x \cdot v / c^2) / \sqrt{\quad}$$

Thus the equation from above becomes

$$(x - v \cdot t) / \sqrt{\quad} = a + w' \cdot (t - x \cdot v / c^2) / \sqrt{\quad}$$

Multiplying both sides by the radical we obtain

$$x - v \cdot t = a \cdot \sqrt{\quad} + w' \cdot (t - x \cdot v / c^2) \quad \text{or}$$

$$x = a \cdot \sqrt{\quad} + v \cdot t + w' \cdot (t - x \cdot v / c^2) = a \cdot \sqrt{\quad} + v \cdot t + w' \cdot t - w' \cdot x \cdot v / c^2$$

From this we obtain

$$x + x \cdot w' \cdot v / c^2 = a \cdot \sqrt{\quad} + v \cdot t + w' \cdot t \quad \text{or} \quad x \cdot (1 + w' \cdot v / c^2) = a \cdot \sqrt{\quad} + (v + w') \cdot t$$

Dividing by the bracketed term on the left we obtain

$$x = a \cdot \sqrt{\quad} / (1 + w' \cdot v / c^2) + (v + w') / (1 + w' \cdot v / c^2) \cdot t$$

Since both the radical and the bracketed term are constant we can read from this that C moves for A with the constant velocity of

$$w = (v + w') / (1 + v \cdot w' / c^2)$$

along the  $x$ -axis!

If we use the symbol  $\oplus$  to represent the relativistic addition of speeds that are parallel to the relative velocity  $v$ , then we can summarize:

$$v \oplus w' = \frac{v + w'}{1 + \frac{v \cdot w'}{c \cdot c}}$$

With the symbol  $+$  we denote the 'usual' addition of numbers. In the numerator we have the usual addition of speeds, while the denominator provides for corrections, as soon as the values of  $v/c$  or  $w'/c$  become substantial. For small speeds of  $v$  and  $w'$  the denominator is practically 1.

In exercise 5 we check that this formula supplies reasonable values in all cases. In the above example of the spaceship with its rocket we get a resulting velocity of

$$0.7 \cdot c \oplus 0.8 \cdot c = (1.5/1.56) \cdot c \approx 0.962 \cdot c$$

## D5 Transverse Velocities and Aberration

How is it for black, when in the red system an object moves with a velocity of  $u'$  transverse to the direction of relative motion  $v$ ? Up to now we have considered only movements along the x-axis.

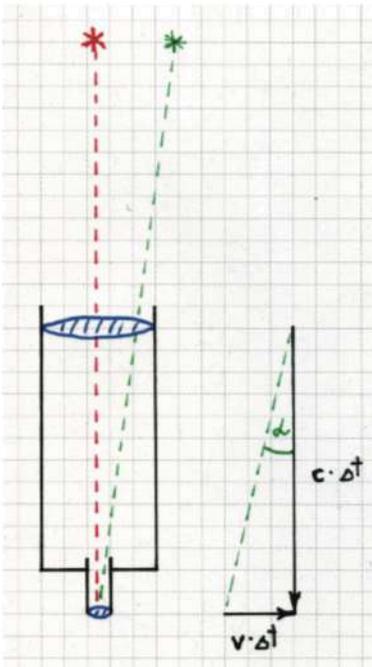
For the derivation of the transformation of such 'transverse velocity' we do not need the Lorentz transformations. Knowledge of the basic phenomena is completely sufficient. Since  $u'$  is a velocity e.g. in the  $y'$ -direction of red, it follows

$$u' = \Delta y' / \Delta t' = \Delta y / \Delta t' = \Delta y / (\Delta t \cdot \sqrt{\phantom{x}}) = (\Delta y / \Delta t) / \sqrt{\phantom{x}} = u / \sqrt{\phantom{x}}$$

Black thus measures the smaller lateral velocity  $u = u' \cdot \sqrt{\phantom{x}}$

We will need this result in **E1**. We use it here for the derivation of the correct formula for aberration. By aberration (lat. aberrare ~ wander, deviate) we understand the change of direction of velocities, which arise as a result of the fact that the viewer likewise moves. James Bradley recognized in 1728 that the tiny annual-periodic position shifts of fixed stars are to be understood as the consequence of the movement of the earth around the sun. According to legend he got the idea, as he rode in his coach in windless rainy English weather and thereby observed that the rain seemed to fall diagonally, the faster the coach moved the more diagonal the rain.

Consider a telescope, pointing in a direction perpendicular to the momentary direction of the earth's movement in its orbit:



In the time the light of a star needs to arrive from the objective to the eyepiece, the earth has already advanced on its course. We must therefore tip the telescope through an angle  $\alpha$ , in order for the star to be presented in the center of the visual field. This angle defines the amount 'the ray of light wanders off' due to the movement of the earth. The resulting formula for this aberration is  $\tan(\alpha) = v/c$ , where  $v$  is the velocity of the earth in its orbit (approximately 30 km/s). The angle has a size of approximately 20 arc seconds.

Einstein had already in 1905 drawn attention to the fact that this traditional formula is only approximately correct. Lateral velocities should be transformed according to the above formula, resulting in the correct formula

$\tan(\alpha) = v/(u' \cdot \sqrt{\phantom{x}}) = v/(c \cdot \sqrt{\phantom{x}})$  One also obtains this formula, if one assigns (correctly) the distance travelled by the light to the hypotenuse of the triangle instead of the leg in the above figure. For the lateral velocity  $u$  of the light, the Pythagorean Theorem gives  $u = c \cdot \sqrt{\phantom{x}}$ , which again yields for  $\tan(\alpha)$  the value  $v/u = v/(c \cdot \sqrt{\phantom{x}})$ . Thus for light the new accurate aberration formula is  $\sin(\alpha) = v/c$ . This correction is astronomically insignificant, since the values of the sine and the tangent functions hardly differ for small angles.

By the way: The angle of aberration is independent of the speed of light in the telescope tube. The angle of aberration does not change if you fill your telescope with water! A more in depth argument would consider the direction of the optical wave planes.

We could now consider the general case, where in the red system an object moves with any velocity  $w' + u'$  in any direction. Einstein already handled this case in his original publication [09-140ff] and presents beautiful symmetrical formulas for the resulting velocities and angles in the black system. Likewise the aberration is treated completely generally [09-146ff]. The appropriate calculations should now be well comprehensible to the reader. We will however not need these results in what follows. **K3** makes some references to it.

## D6 The Optical Doppler Effect

You all know the phenomenon: An ambulance approaches at high speed with howling siren. As it races by the pitch of the siren sinks and remains constantly at a deeper level as it departs. Also the other situation is well well-known, where you yourself move at high speed past a standing source of noise: You are travelling in your car over country, windows open, and you past a train crossing with alarm bells.

These changes in the perceived pitches correspond to measurable changes in the frequencies of the acoustic waves. Christian Doppler examined this theoretically and concluded that the two cases "listener moves, source at rest" and "source moves, listener at rest" must differ. In 1842 he ascribed formulas, indicating how the measured values of the frequencies and wavelengths change. Today one calls the phenomenon in his honor the "Doppler effect". We can easily understand why one may not only consider the relative motion of source and listener *acoustically*: The sound spreads out homogeneously with a certain speed in all directions *in the medium of air* ! This carrier medium supplies a special inertial frame and naturally served as the model for the ether, in which the light should spread.

Consider an observer B with velocity  $v$  approaching a resting acoustic source Q. This produces a tone of frequency  $f(Q)$ . What frequency  $f(B)$  does the observer measure? Doppler's answer to this question is the following, whereby here  $c$  means the speed of sound in air:

$$(1) \quad f(B) = f(Q) \cdot (1 + v/c) \quad \text{Observer approaches a resting acoustic source}$$

For the case where the acoustic source approaches with velocity  $v$  an observer at rest in the medium air, we have

$$(2) \quad f(B) = f(Q) / (1 - v/c) \quad \text{Acoustic source approaches a resting observer}$$

For values of  $v/c < 0.1$  the two results hardly differ. The difference becomes arbitrarily large however as  $v/c$  approaches the value 1.

We change now from sound to light or more generally to electromagnetic waves. The STR precludes the possibility of determining absolutely, who is at rest and who is moving. Thus 'optically' there is only one Doppler formula! We deduce it first from (2):

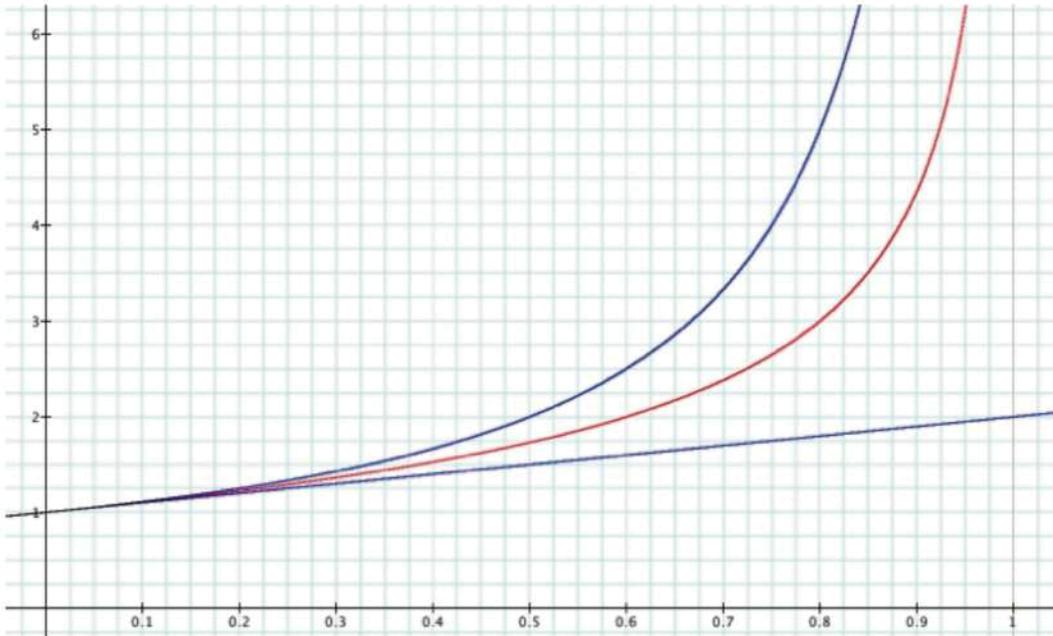
Formula (2) remains valid, but we must now consider additionally that the oscillator of the transmitter, because of time dilation, oscillates for the observer B only with the frequency  $f(Q) \cdot \sqrt{1 - v/c}$  ! Thus we have  $f(B) = f(Q) \cdot \sqrt{1 - v/c} / (1 - v/c)$ . Considering that we can write  $\sqrt{1 - v/c} = \sqrt{(1 + v/c) \cdot (1 - v/c)}$ , we can reduce a little obtaining

$$f(B) = f(Q) \cdot \sqrt{\frac{1 + v/c}{1 - v/c}} = f(Q) \cdot \sqrt{\frac{c + v}{c - v}}$$

We obtain the same result, if we argue from the view of the source at rest and proceed from formula (1). The receiver B then counts more oscillations with his slowed clock, i.e.  $1/\sqrt{1 - v/c}$  times as many per second as someone whose clocks do not tick more slowly. Thus  $f(B) = f(Q) \cdot (1 + v/c) / \sqrt{1 - v/c}$ , which after simplifying provides the same result above.

Both Doppler formulas provide for light in the STR the same frequency shift and the cases 'source at rest' and 'observer at rest' can no longer be differentiated.

Let us graph the three functions  $y = 1 + x$ ;  $y = 1/(1 - x)$  and  $y = \sqrt{((1 + x)/(1 - x))}$  with the value of  $x = v/c$  ranging over the unit segment 0 to 1. We have the following picture:



The lower, linear function belongs to Doppler formula (1), the upper blue to Doppler formula (2). The middle red curve describes the optical (or relativistic) Doppler effect in accordance with the formula deduced previously. The differences begin to be evident only when  $v/c$  is larger than about 0.2. Starting from a value of  $v/c$  greater than 0.5 the differences become increasingly dramatic.

In astronomy the optical Doppler effect has important applications. Frequencies of spectral lines, however, are not measured but rather the wavelengths (usual symbol  $\lambda$ ). Therefore we should transform the above formula accordingly:

In general  $\lambda \cdot f = c$  or  $f = c / \lambda$ . Thus

$$c / \lambda(B) = (c / \lambda(Q)) \cdot \sqrt{(c + v) / (c - v)} \quad \text{and after division by } c$$

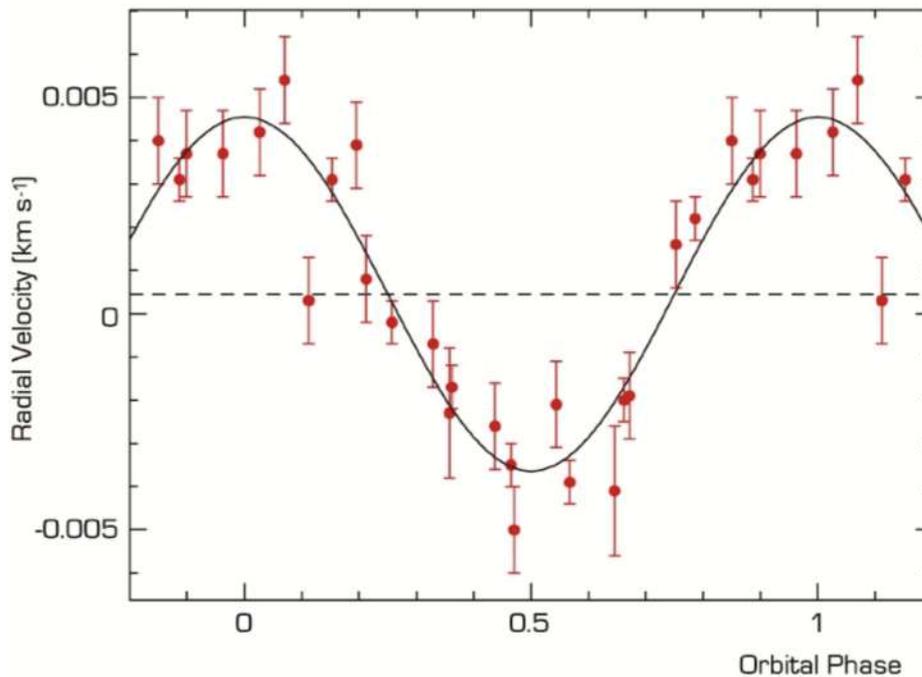
$$\lambda(Q) = \lambda(B) \cdot \sqrt{(c + v) / (c - v)} \quad \text{or} \quad \lambda(B) = \lambda(Q) \cdot \sqrt{(c - v) / (c + v)}$$

$\lambda(Q)$  is well-known and  $\lambda(B)$  is measured. From this the velocity  $v$  can be computed, with which the source moves toward us ( $v > 0$ ) or away from us ( $v < 0$ ). This is the so-called radial velocity. If one solves the above formula for  $v$ , then one obtains

$$v = c \cdot \frac{\lambda(Q)^2 - \lambda(B)^2}{\lambda(Q)^2 + \lambda(B)^2}$$

In the last few years astronomers have developed such precise spectrometers that they can measure periodic fluctuations in the radial velocity of stars within the range of a few meters per second. This has become one of the most important methods for showing the existence of planets orbiting other stars (so-called exoplanets). The graphic below gives an impression of the precision that has been obtained. The measured values of the radial velocity have an uncertainty of approximately  $\pm 1$  m/s! These fluctuations of the radial velocity result from the fact that both the planet *and* the star orbit a common center of mass.

Further information is freely available on the well maintained web page of the ESO. The following graphic was taken from the ESO press release of August 25, 2004. Consult the web-site for an answer to the question, how long does an 'orbital phase' last, in other words, how long is the period of this planet in days or hours.



"Velocity Curve" of mu Arae

ESO PR Photo 25d/04 (25 August 2004)

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<http://www.eso.org/outreach/press-rel/pr-2004/pr-22-04.html>

## D7 Problems and Suggestions

1. A fighter jet flies with 1000 m/s and shoots a projectile off in its flight direction with a muzzle velocity of likewise 1000 m/s. Add these velocities 'classically' and 'relativistically'.
2. Derive the Lorentz transformations for  $t'$  and  $x'$  algebraically from those for  $t$  and  $x$ , which we deduced first in **D2** and then a second time in **D3**. Why is that actually unnecessary?
3. Show algebraically that the Lorentz transformations from the non-prime to the prime system and vice-versa mutually cancel each other.
4. Derive the Lorentz transformations for  $t'$  and  $x'$  from an Epstein diagram!
5. Examine our formula from **D4** for relativistic velocity addition. Are  $v = 0.5 \cdot c$ ,  $w = 0.8 \cdot c$ ,  $u = -0.5 \cdot c$  and  $c$  all parallel velocities. Form a)  $v \oplus v$  b)  $v \oplus w$  c)  $v \oplus c$  d)  $c \oplus w$  e)  $c \oplus c$  f)  $c \oplus u$  g)  $u \oplus -c$  h)  $w \oplus w$
6. How quickly is a star approaching us, given that the H $\alpha$  line for an excited hydrogen atom is not found to be 656 nm as in a laboratory on earth, but rather at 649 nm? (The emission line is thus a little 'blue' shifted).
7. How fast does one have to approach a traffic light, so that one sees the red light (wavelength of 620 nm) as green (wavelength of 520 nm)?
8. A laser produces light at 632 nm wavelength. What wavelength do we measure, if this laser is at the tail of a UFO, which is moving away from us at  $0.5 \cdot c$ ?
9. Why does the rotation of a star show up as a widening of its spectral lines?
10. Why do spectral lines widen, if an emitting gas exhibits a high temperature and high pressure? (The effects in problems 9 and 10 express themselves quantitatively differently and can be partly computationally separated, if they arise superimposed.)
11. Derive the optical Doppler formula from the acoustic Doppler formulas for the wavelengths, by additionally considering length contraction!
12. Read pages 140-142 as well as 146-149 from Einstein's original publication in [09].
13. In addition to our 2 coordinate systems (black and red with the points A and B) there is a 'middle' system C, in which A moves equally fast to the left as B to the right. Ascertain, in general, the velocity of this middle system C for both A and B. Without STR the answer would naturally be  $v/2$  and  $-v/2$ ...  
The existence of this middle system C, by the way, provides a beautiful argument for the fact that the relative velocities of B for A and of A for B must be quantitatively equal: From the point of view of C the situation is perfectly symmetrical!
14. How do the radar speed measurements of the traffic police function? Consider the whole from the frame of the reflecting car.

II. Prüfungsergebnisse.							
Schulprüfung.				Turnprüfung.			
Jahr 19 _____ Kontr.-Nr. _____				Jahr 19 _____ Kontr.-Nr. _____			
Lehrer:	Adress:	Notizen:	Teilnahme:	Wettkamp:	Notizen:	Teil:	Teil:
Der Sekretär der Prüfungskommission: <b>Kreiskommando Zürich</b>				Der Sekretär der Turnprüfungskommission: <b>Kreiskommando Zürich</b>			
III. Sanitarische Untersuchungen.							
1. Befund der Untersuchungskommission.							
Div.- u. Rekrutierungskreis <i>W/6</i>				Untersuchungskontrolle Nr. <i>1204</i>			
Körperlänge <i>171,5</i> cm.				Oberarm <i>28</i> cm.			
Brustumfang <i>87</i> cm.				Sehschärfe <i>1/200</i>			
Ekrankheiten oder Gebrechen:							
<i>Varices, Pla. planus</i>							
<i>Hyperidrosis ped.</i>							
Verfügung der Untersuchungskommission:							
<i>Unbrauchbar A.</i>							
<b>ZÜRICH</b> , den <i>13. März 1901</i>							
Der Sekretär der Untersuchungskommission: <b>Kreiskommando Zürich</b>							
*) Stempel des Divisionsarztes.							

Albert Einstein's Swiss Military "Service Booklet". He was declared exempt from service when found to have flat and sweaty feet - certainly to his great satisfaction.

Einstein fled from Munich not only because he found the prevailing spirit at the Luitpold Gymnasium unbearable. He also feared being conscripted by the military - an idea that certainly filled him with horror and persuaded him to abandon German citizenship. He deeply hated everything militaristic, and repeatedly supported conscientious objectors and lobbied his entire life for disarmament and the strengthening of supranational institutions.

"This topic brings me to that worst outcrop of the herd nature, the military system, which I abhor. That a man can take pleasure in marching in formation to the strains of a band is enough to make me despise him. He has only been given his big brain by mistake; a backbone was all he needed. This plague-spot of civilization ought to be abolished with all possible speed." [20-6]

Yet Einstein was no naive pacifist. In view of what was brewing in the early thirties in Germany on his Berlin doorstep, he forsook his previous strictly pacifist line and wrote to a Belgian military objector:

"Organized power can be opposed only by organized power. Much as I regret this, there is no other way." [17-168]



The Sombrero Galaxy (VLT ANTU + FORS1)

ESO PR Photo 07a/00 (22 February 2000)

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## E Mass, Momentum and Energy

We now turn from kinematics to dynamics. In the first section we examine in the context of an elegant thought experiment, what influence a relative motion has on 'mass' and 'momentum'. This provides a new argument for the fact that  $c$  is a limiting velocity. We also introduce a beautiful Epstein diagram for mass and momentum, which corresponds to the one for space and time. We then turn to the meaning of energy and recognize that energy must have inertial mass. But how much mass does a given quantity of energy have? A calculation provides the best known formula of all of physics. This result can also be presented in an Epstein diagram using the sizes of energy and momentum.

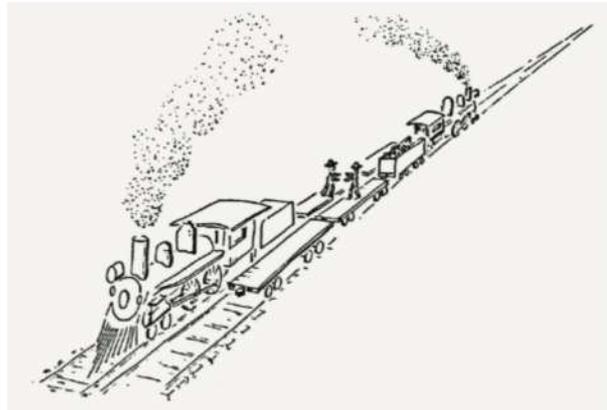
## E1 The Symmetric Punch

The English and the French were for many years on the warpath with one another. They not only fought politically and militarily, but they also argued about whether the 'punch' (lat. 'impetus') of a projectile increased linearly with speed or as the square of the speed. On the island a vector-oriented viewpoint was preferred, and accordingly 'momentum' was described by  $p = m \cdot v$ . On the continent one rather relied on scalar values such as  $E = 0.5 \cdot m \cdot v^2$ . These preferences still show up today in colloquial language: If an Englishman must determine a value 'then he will figure it out' (tending to draw a geometric figure), while the Frenchman 'va calculer ça' (tending to solve some algebraic equations).

Today we know that both momentum and the kinetic energy have meaning, and therefore it does not surprise us that both the French and the English got correct results. This example shows very nicely, how the physical terminology needed to gradually crystallize and did not already possess their current definitiveness when they were first introduced. It is precisely these terms, i.e. (inertial) mass, momentum and kinetic energy, which are the subject of this chapter.

We must first speak of momentum, *which we maintain is defined by*  $\mathbf{p} = m \cdot \mathbf{v}$  ! As soon as it is clarified how inertial mass is transformed from one reference system to another, then it is also clear what happens to the momentum, because we already know the transformation of the velocities in **D4** and **D5**. In any case it should be clear that momentum, just like velocity, is a highly relative quantity whose value (not only in the STR !) depends entirely on the choice of reference system: In the system of a body at rest its momentum is always zero.

Our derivation follows the beautiful presentation in [15-110ff]. This presentation is beautiful because it deduces the equality of two values based on the symmetry of a thought experiment. The twins Peter and Danny are each on one of two Einstein trains, which are racing past each other in opposite directions on a long straight stretch of track. They each throw completely symmetrical punches at each other perpendicular to the direction of travel of the trains:



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The symmetry of the situation does not allow one twin to punch more strongly than the other. Both fists (measured at rest!) are of equal weight and both fists are equally fast in their own system ( $u' = -w$ ). At any time the sum of the impulses in the y-direction, that is, perpendicular to the direction of travel, is zero for both. We regard the impact with Peter in the black, non-prime system and see the perpendicular velocity of  $u'$  of Danny's fist slowed according to **D5**. That Danny's fist carries nevertheless an equally large momentum as Peter's can only be explained in that Danny has somehow increased the mass of his fist. Here Epstein tells the story of the aging bouncer in Bourbon Street, who can no longer punch as fast as former times and compensates by putting a role of coins into his fist, in order to give his punch its former strength...

We soberly calculate assuming that the mass depends on the relative velocity. We use  $v$  (as always) to show the relative velocity in the x-direction, that is, the direction of travel of the trains:

$\mathbf{p}_y$ (Danny) = - $\mathbf{p}_y$ (Peter)	due to symmetry for Peter and for Danny
$m_{v+u} \cdot \mathbf{u} = -m_w \cdot \mathbf{w}$	for Peter and us! $\mathbf{u} = \mathbf{u}' \cdot \sqrt{\phantom{x}} = -\mathbf{w} \cdot \sqrt{\phantom{x}}$ (according to D5) yields
$m_{v+u} \cdot (-\mathbf{w} \cdot \sqrt{\phantom{x}}) = -m_w \cdot \mathbf{w}$	dividing through by $-\mathbf{w}$ we obtain
$m_{v+u} \cdot \sqrt{\phantom{x}} = m_w$	where $\mathbf{v}$ and $\mathbf{u}$ are perpendicular and are added as vectors!

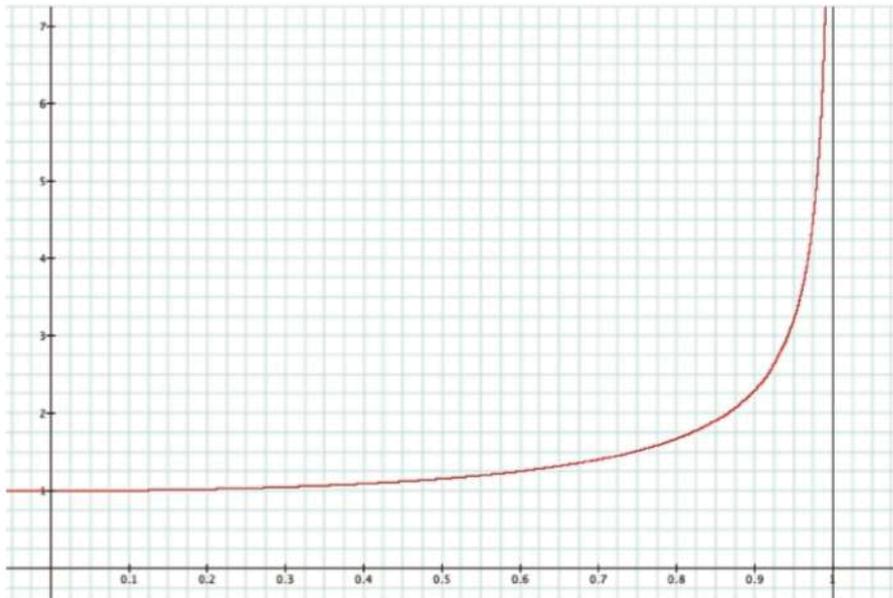
This relationship applies for arbitrarily small perpendicular speeds of  $\mathbf{u}' = -\mathbf{w}$ , and also for the limiting case of  $\mathbf{u}' = -\mathbf{w} = 0$ . In that case  $u = 0$ , and we can write  $m_{v+u}$  as  $m_v$ . The term  $m_w = m_0$  represents the mass of the fist at rest, the so-called *rest mass*. Thus we have found the dependence of the mass on the relative velocity  $v$ : The inertial mass of a body increases with the relative velocity, giving

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This 'dynamic mass'  $m_v$  is also used for the relativistic momentum:

$$\mathbf{p} = m_v \cdot \mathbf{v} = m_0 \cdot \mathbf{v} / \sqrt{\phantom{x}}$$

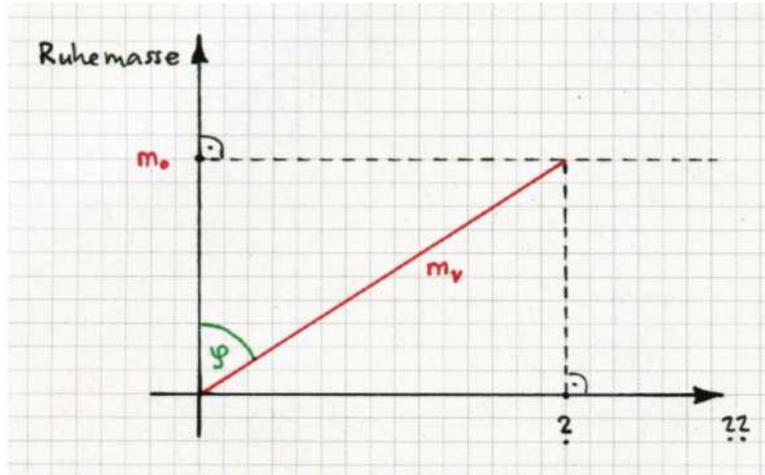
Thus the mass of a particle increases with its velocity. Without allowing for this relativistic effect modern particle accelerators could not function! The mass increase is dramatic when the particle approaches the speed of light: The relationship  $m_v/m_0$  follows the function  $1/\sqrt{\phantom{x}}$ , and as  $v \rightarrow c$  this function approaches infinity:



We now have a completely new reason that  $c$  is a limiting velocity: The mass being accelerated increases infinitely, as the velocity  $v$  approaches the speed of light! The body thus offers an ever larger resistance to further acceleration. We will see this even better when we treat the question of energy.

## E2 Epstein Diagrams for Mass and Momentum

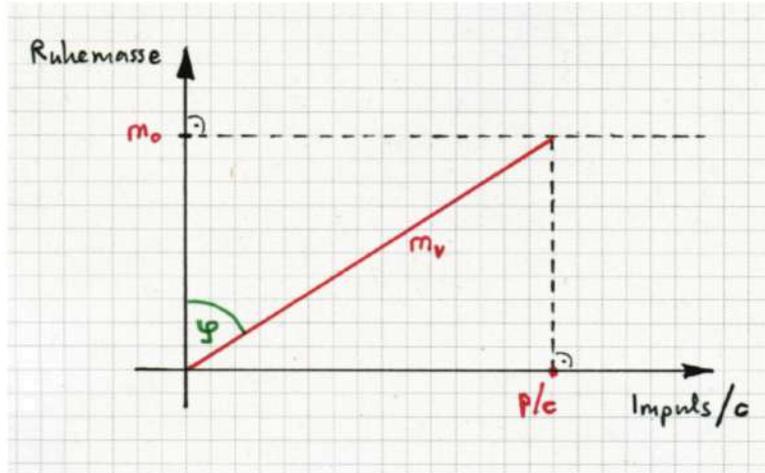
Thus we obtain 'dynamic mass'  $m_v$  by dividing 'rest mass'  $m_0$  by our well-known radical, which corresponds in the Epstein diagram to the  $\cos(\phi)$ . Therefore we can represent  $m_v$  and  $m_0$  in a simple diagram (Note: German 'Ruhemasse' means 'rest mass'):



Does the projection of  $m_v$  on the horizontal axis have a meaning and if so what is it? We obtain this projection with  $m_v \cdot \sin(\phi)$ , and if we remember what the sine value means in the Epstein diagram, then we immediately have

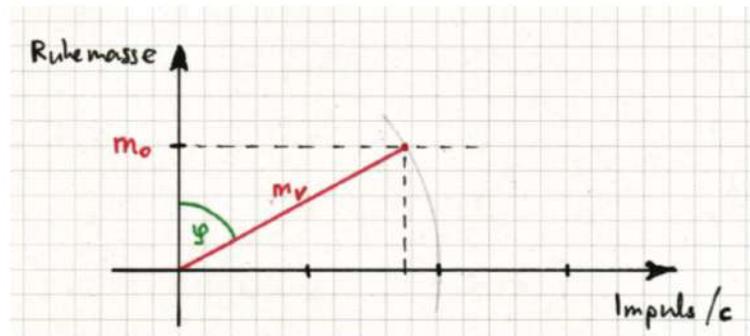
$$\langle ? \rangle = m_v \cdot \sin(\phi) = m_v \cdot v / c = p / c$$

Thus we can completely label the Epstein diagram for mass and momentum (Note: German 'Impuls' means 'momentum'):



The faster an object is the larger the angle  $\phi$  becomes, and the more the dynamic mass and the momentum of the object grow (if the rest mass remains constant).  $\sin(\phi)$  and  $\cos(\phi)$  keep their former meaning. Here one sees again, what a beautiful representation we would have if  $c$  simply had the unit-less value 1: one space-time unit per space-time unit. The transition from the usual technical units to these more 'natural' ones is, however, easy to make and is open to the willing reader. We nevertheless permit ourselves to speak of this diagram as the mass-momentum diagram.

We solve yet another standard task with such a mass-momentum diagram: How fast does an object have to move in order to double its mass? If we give  $m_0$  5 little squares then  $m_v$  amounts to 10 little squares:

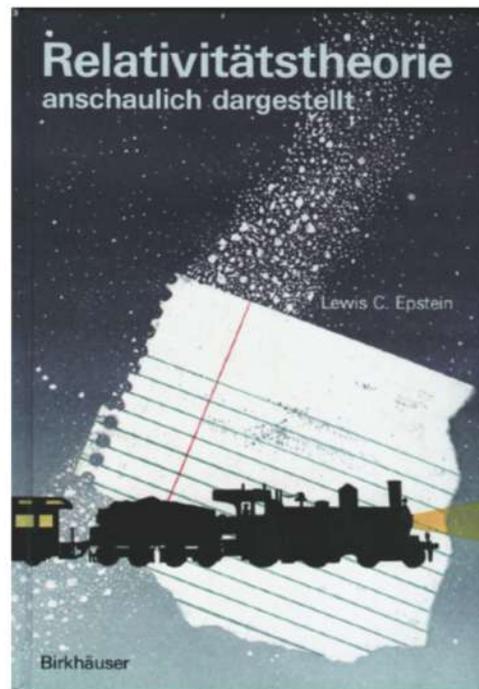
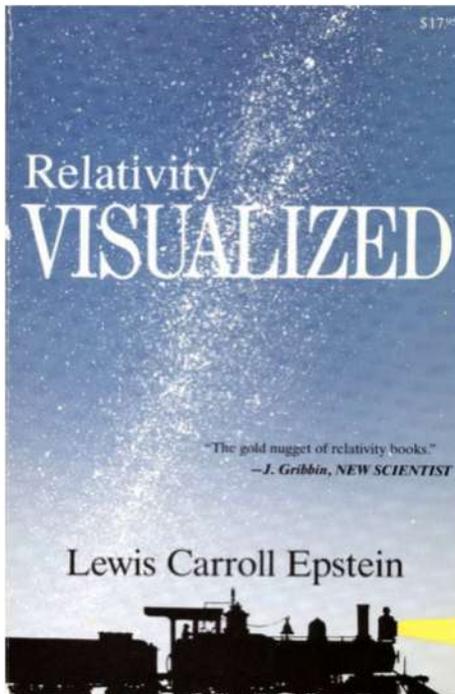


A compass arc with radius 10 squares gives us  $m_v$  and  $\phi$ . We read from the diagram:

$$v/c = \sin(\phi) \approx (8.6 \text{ or } 8.7 \text{ squares}) / (10 \text{ squares}) \approx 0.86 \text{ or } 0.87$$

The object must therefore move with approximately 87% of the speed of light. The calculation of  $v/c$  yields the exact value  $\sqrt{3}/2$  with the approximate value 0.8660.

The opposite question ('How large is  $m_v/m_0$  for an object, which moves with 90% of  $c$ ?) can be answered just as easily with an Epstein mass-momentum diagram. If one wants to completely avoid using a pocket calculator then one should make certain that  $m_v$  in the denominator corresponds to a simple number of squares (best 10 or 20). For three or more digits of accuracy then millimeter scaled graph paper must be used ...



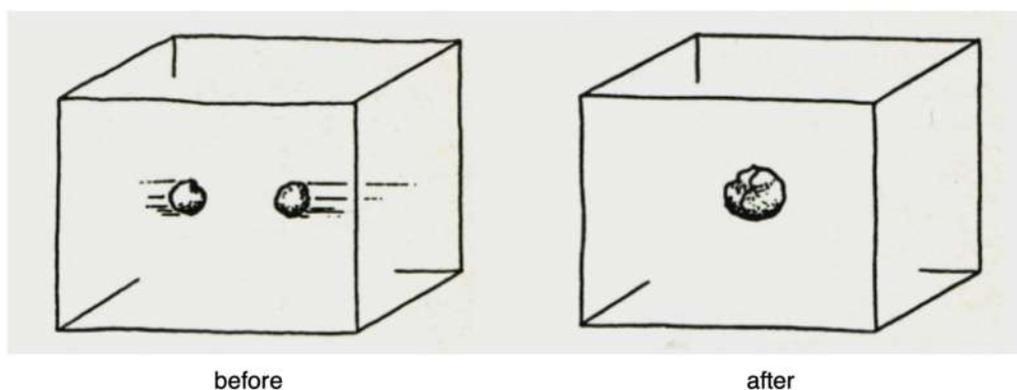
Samuel Edelstein's copy of [15] and David Eckstein's copy of the German edition

Although recommended by many authors, both the English and the German editions of the book are out of print. A few used copies are available at rather high prices via Amazon.

### E3 Mass and Energy – Observations in a Closed System

For the following considerations we must introduce the concept of a 'closed system'. One imagines an arbitrarily large, clearly limited space (e.g. a cube, a box, or the inside of an enormous thermos bottle etc.). One postulates that the enclosed area has no exchange with the surrounding space: Neither matter nor charge, neither energy nor momenta flow through the walls. No fields on the outside have influence on the inside nor vice-versa, nor do any forces from the outside act on the inside, nor any from the inside on the outside. We imagine such an ideal region of space and we immediately admit that no such thing really exists. We also know that no Einstein trains exist, no ideal clocks and no rigid yardsticks! That cannot prevent from imagining such things.

Let such a closed system contains only two lead clumps of equivalent mass, which race toward each other with equal velocities. After the collision they form one large clump of lead at rest (diagram and argument from [15-117]):

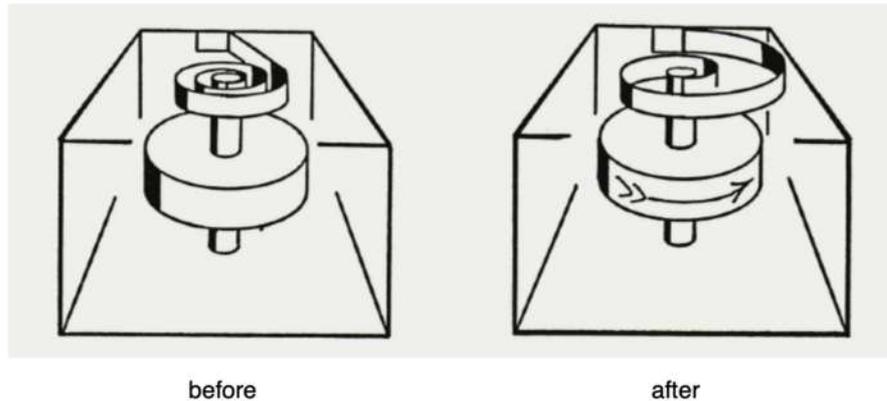


The drawings on this and on the next side are Copyright protected © 2000 by Lewis C. Epstein, *Relativity Visualized*

Where does the increased mass go, which, due to their speed, the two clumps possess before the collision? It must be somewhere in our closed system. Students usually hit upon a good answer: The large resting clump is warmer due to the deformation of the collision. The individual particles of matter have an increased velocity. The dynamic mass is thus still available, but no longer macroscopically visible. Okay, but that means that we add mass to a stone, whenever we warm it up - no matter how -! Energy input is thus connected to an increase in mass. If we use a car battery and a radiating heater, in order to warm up a stone, the stone will have more mass afterwards than before - and the battery less! With the flow of energy from the battery into the stone we have also moved mass into the stone!

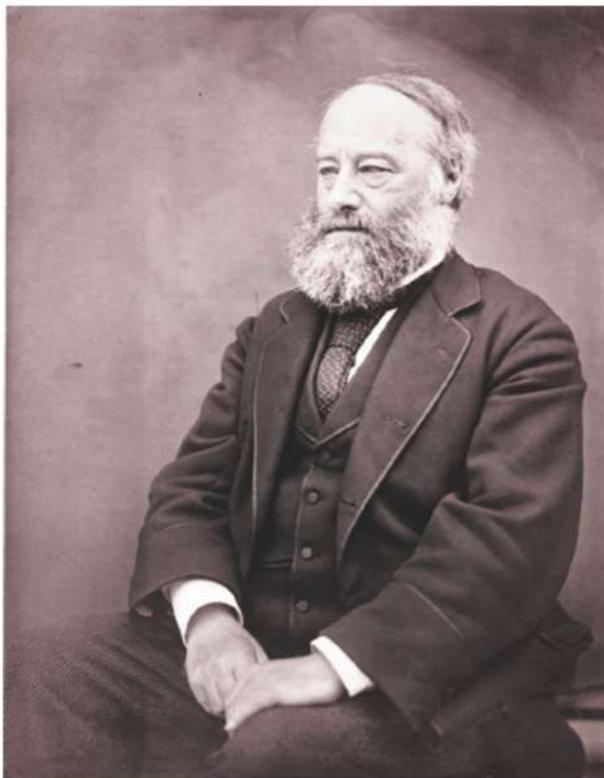
We can easily switch the thermodynamics off in our thought experiment, if we reverse the process: A spring is wedged between two small clumps with the assistance of a thread. The thread is strained to its limit and is about to break. The spring relaxes, remains where it was and the two clumps race in opposite directions from it. They both have a large speed now and their mass thus increases. Where does this mass come from? The spring got warmer when relaxing, so the extra mass cannot this time be from the temperature. If afterwards the clumps have more mass - what did we have before? Before elastic energy was in the tensed spring, and the additional mass must have come from it.

Epstein provides in [15-117f] a similar example:



*Before*, the flywheel is at rest and the spiral spring is tensed. *After*, we have a relaxed spiral spring and a rotating flywheel. The rotating flywheel must have more mass than the one standing still. This additional mass can come only from the energy in the tensed spring. Yes, we must say that this additional mass of the flywheel was previously in the tensed spring, if we assume the entire mass in a closed system must be constant.

We must get accustomed to the idea that an energy input always implies an increase of mass. A spring has more mass after stretching than before, and a loaded condenser must have more mass than one unloaded, although only some electrons were shifted from one condenser plate to the other. The question then arises, how much additional mass does a joule of additional energy bring? The Scottish brewer and 'amateur physicist' James Prescott Joule answered the question in 1843, how many joules of mechanical energy correspond to a calorie of heat energy. We must now clarify, how many joules of energy correspond to a kilogram of mass!



Joule at the age of 16 to 18 enjoyed together with his brother two years of private instruction with the great John Dalton. He was not allowed to study at a university and had to take over the direction of the family brewery at an early age. It is very informative to see how hesitantly the distinguished Royal Society in London and other established gentlemen took notice of Joule's elegant experiments. Commendable exceptions were James Clerk Maxwell, whom we have already met and John Davis, promoter also of another great autodidact Faraday.

Read the wikipedia contribution to Joule!

James Prescott Joule (1818-1889)

## E4 Energy has Mass. How Much Mass Does 1 Joule Have?

If we accelerate an object with rest mass  $m_0$ , it gains not only velocity and kinetic energy, but its mass  $m_v$  also becomes larger. We want to now treat this computationally, in order to determine quantitatively the additional mass gained by this energy input. We want to exactly derive this important result and therefore need integral calculus, albeit only to the extent that a first year calculus student has at their disposal. Physics can also serve mathematics by showing how powerful formal mathematical methods can be.

The kinetic energy is equal to the accumulated work of acceleration, and we obtain this with the integral  $F \cdot ds$  over the distance of the acceleration:

$$F \cdot ds = \frac{d(m \cdot v)}{dt} \cdot ds = \frac{d(m \cdot v)}{dv} \cdot \frac{dv}{dt} \cdot ds = \frac{d(m \cdot v)}{dv} \cdot \frac{ds}{dt} \cdot dv = \frac{d(m \cdot v)}{dv} \cdot v \cdot dv$$

$\Delta W = \int F \cdot ds = \Delta E_{\text{kin}} = E_{\text{kin}}$  (the last equal sign pertains only to the case where  $v_0 = 0$ )

According to Newton force  $F$  is the change of momentum over time:  $F = dp / dt = d(m \cdot v) / dt$

We use this to rewrite the integral over  $F \cdot ds$ :

Instead of integrating over the distance of acceleration we can now integrate over the increase in velocity

$$\Delta W = \int_{s_0}^{s_{\text{end}}} F \cdot ds = \int_{v_0}^{v_{\text{end}}} \frac{d}{dv}(m \cdot v) \cdot v \cdot dv = \int_0^{v_{\text{end}}} \frac{d}{dv}(m \cdot v) \cdot v \cdot dv = E_{\text{kin}} \quad (1)$$

In order to gain some confidence in this procedure we first compute the classical case. In this case the accelerated mass is constant, and the derivative of  $m \cdot v$  over  $v$  simply gives  $m$ . Thus (1) yields:

$$\Delta W = \int_0^{v_{\text{end}}} \frac{d}{dv}(m \cdot v) \cdot v \cdot dv = \int_0^{v_{\text{end}}} m \cdot v \cdot dv = m \cdot \int_0^{v_{\text{end}}} v \cdot dv = m \cdot \left[ \frac{1}{2} \cdot v^2 \right]_0^{v_{\text{end}}} = \frac{1}{2} \cdot m \cdot v_{\text{end}}^2 = E_{\text{kin}}$$

We obtain the familiar expression for kinetic energy, which hopefully somewhat alleviates any distrust arising from all the juggling of  $dv$ 's and  $dt$ 's!

What value does the expression  $d(m \cdot v) / dv$  have in the relativistic calculation? It is

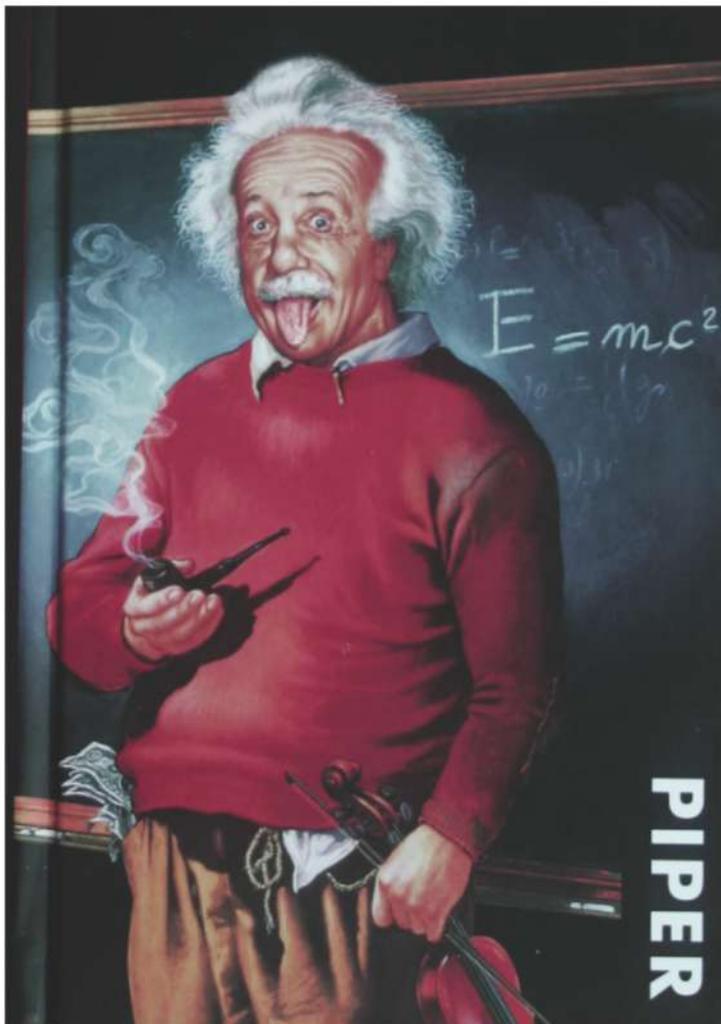
$$\frac{d}{dv}(m \cdot v) = \frac{d}{dv} \left( \frac{m_0 \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = m_0 \cdot \frac{d}{dv} \left( \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = (\dots \text{rechne} \dots) = m_0 \cdot \left( 1 - \frac{v^2}{c^2} \right)^{-3/2}$$

This expression is occasionally called the 'longitudinal mass'. We stress however that there is only one expression for the inertial mass of a body, i.e.  $m_v = m_0 / \sqrt{1 - v^2/c^2}$ , and that expression is direction-independent. People spoke of the longitudinal and transversal mass before clarifying that force and acceleration transform differently for the directions parallel and perpendicular to  $v$ .

Thus we can also calculate (1) in the relativistic case:

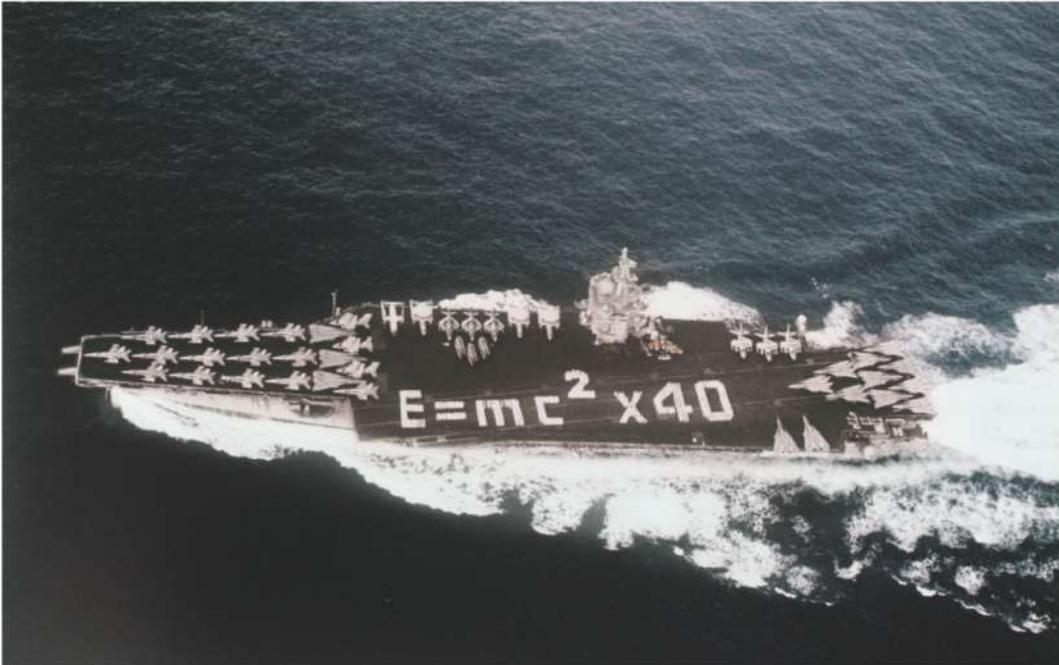
$$\begin{aligned}
 \Delta W &= \int_0^{v_{\text{end}}} m_0 \cdot \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \cdot v \cdot dv = m_0 \cdot \int_0^{v_{\text{end}}} \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \cdot v \cdot dv = && m_0 \text{ being a constant term} \\
 &= m_0 \cdot \frac{c^2}{-2} \cdot \int_0^{v_{\text{end}}} \frac{-2 \cdot v}{c^2} \cdot \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \cdot dv = && \text{write the 'inner derivation' as an explicit factor} \\
 &= m_0 \cdot \frac{c^2}{-2} \cdot (-2) \cdot \left[ \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \right]_0^{v_{\text{end}}} = && \text{simplify and insert limits} \\
 &= m_0 \cdot c^2 \cdot \left[ \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1 \right] = && \text{expand and remember the} \\
 &= m_v \cdot c^2 - m_0 \cdot c^2 = \Delta m \cdot c^2 && \text{the definition of } m_v \text{ in E1 !}
 \end{aligned}$$

We have found the connection between the energy gain  $\Delta E$  (or performed work  $\Delta W$ ) and the mass increase  $\Delta m$  we were looking for. The resulting formula is so simple that it has attained a strange popularity.



The illustration on the cover of the otherwise smart booklet [21] about the STR serves all the popular clichés. Of course, the formula  $E = mc^2$  is not missing ...

The US Navy even improved the formula in celebration of the 40th anniversary of nuclear-powered aircraft carriers. Sailors served as 'pixels':



Back to physics. We present in a red box, what Einstein himself later designated as the most meaningful result of the STR:

$$\Delta E = \Delta m \cdot c^2$$

Thus 1 Joule of energy produces an increase in mass of around 1 kg divided by  $c^2$ . Here we are glad that we did not standardize  $c$  with the value 1, since otherwise this conversion factor between the energy and the mass would not be so obvious!

Our derivation also reveals the correct expression in the SRT for kinetic energy:

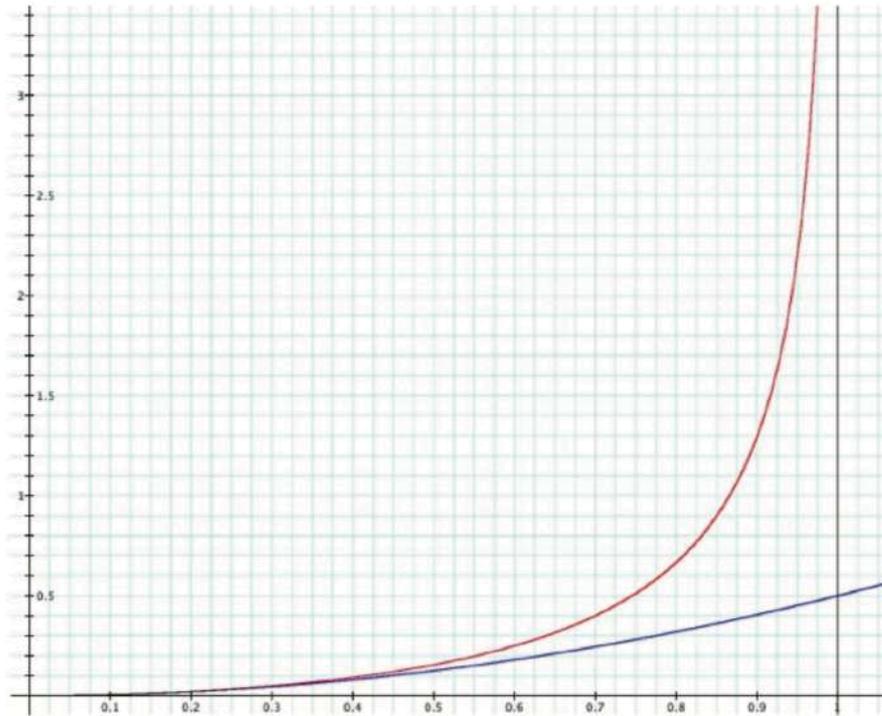
$$E_{\text{kin}} = (m_v - m_o) \cdot c^2$$

That this formula turns into the classical expression  $0.5 \cdot m_o \cdot v^2$  for small speeds is not obvious. If the graphic on the following page does not suffice, then try the following: Develop the term  $1/\sqrt{1 - (v/c)^2} = (1 - x^2)^{-1/2}$  for  $x$  into a power series (find a collection of formulas or use a computer algebra program) and cancel out (for small values of  $x$ ) the fourth and higher order terms.

By the way, Einstein found the relationship  $\Delta E = \Delta m \cdot c^2$  shortly after the appearance of [09-123ff] and in the autumn of 1905 communicated it quasi as an addendum [09-161ff]. As early as 1901 Walter Kaufmann (1871-1947) had pondered a dependence of 'transversal mass' on velocity due to measurements he made of fast moving electrons. Due to its fundamental meaning the formula was experimentally examined again and again. In 2005 two groups of researchers in Canada and the USA were able to increase the accuracy to 1 part per million [nature 438, p.1096-1097].

And, above all: None of the many experiments could prove any deviation from Einstein's formula! Theories cannot be confirmed by experiments, however they can be falsified.

We want to compare the kinetic energy according to the classical and the relativistic calculation in a diagram. We draw  $E_{kin} / E_0$  for values  $x = v/c$  from 0 to 1. The classical behavior corresponds to the blue curve with  $y = 0.5 \cdot x^2$ , while the relativistic to the red curve with  $y = 1/\sqrt{1 - x^2} - 1$  :



The curves deviate from each other only for larger velocities. Electrons can readily be accelerated to  $0.8 \cdot c$  and thereby show clear deviations from classical behavior (experiments by Kaufmann, see problem 4).

Very fast-moving particles ( $v \approx c$ ) also make possible a very fast derivation of the relationship between the mass increase and the energy gain. Assume a particle already has a velocity, which differs only fractionally from  $c$  (e.g., by a thousandth of  $c$ ). Any added energy will serve practically only to increase the mass. As a good approximation  $p = m_v \cdot v \approx m_v \cdot c$  and thus  $dp / dv = c \cdot dm / dv$ . It follows immediately

$$dW = (dp / dv) \cdot v \cdot dv = c \cdot (dm / dv) \cdot c \cdot dv = c^2 \cdot dm$$

and we are finished: The mass increase is proportional to the energy input, and the proportionality factor is the square of the speed of light! This is actually the counterpart to the classical calculation, which we made at the beginning of this section and where we additionally assumed  $m$  to be constant.

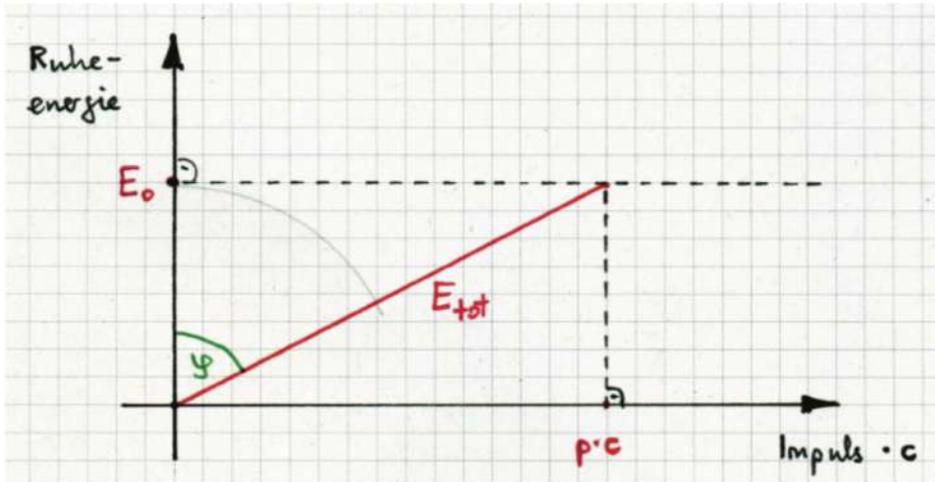
Thus the product of a mass with the square of a velocity represents (as we knew all along) an energy. We derived:

$$\Delta W = m_v \cdot c^2 - m_0 \cdot c^2 = \Delta m \cdot c^2 = E_{kin} = m_0 \cdot c^2 \cdot (1 / \sqrt{1 - x^2} - 1)$$

The expression  $m_0 \cdot c^2$  represents the quantity of energy corresponding to the rest mass, which the object already possessed before acceleration. Therefore one calls  $m_0 \cdot c^2$  the *rest energy* of the object and writes it as  $E_0$ . The expression  $m_v \cdot c^2$  stands for the sum of the rest energy and the kinetic energy and is therefore the total energy of the object. We will write this as  $E_{tot}$ .

## E5 Epstein Diagrams for Energy and Momentum

Why not make our lives easy for once? Take the Epstein diagram for mass and momentum of **E2** and multiply all distances by  $c^2$ ! After a close look we now see that we have an Epstein diagram for energy and momentum before us: The rest mass  $m_0$  becomes  $m_0 \cdot c^2$ , and therefore the rest energy  $E_0$ , the dynamic mass  $m_v$  becomes  $m_v \cdot c^2$ , and therefore the total energy  $E_{tot}$ , and on the horizontal axis instead of  $p / c$  we have  $p \cdot c$ :



Naturally  $\phi$ ,  $\sin(\phi)$  and  $\cos(\phi)$  keep their previous meaning. We can however check the relations in this new context:

$$E_{tot} \cdot \sin(\phi) = m_v \cdot c^2 \cdot v / c = m_v \cdot c \cdot v = m_v \cdot v \cdot c = p \cdot c$$

$$E_{tot} \cdot \cos(\phi) = m_v \cdot c^2 \cdot \sqrt{1 - v^2/c^2} = (m_v \cdot \sqrt{1 - v^2/c^2}) \cdot c^2 = m_0 \cdot c^2 = E_0$$

We also maintain the important relationship between the energies and momentum. Using the Pythagorean Theorem:

$$(E_{tot})^2 = (E_0)^2 + (p \cdot c)^2 \quad (2)$$

Also kinetic energy can easily be made visible: One circumscribes a circle around the origin with radius  $E_0$  and notes that  $E_{kin}$  is the difference  $E_{tot} - E_0$ .

We should again point out that no particle that has a non zero rest mass can ever completely reach the speed of light. One would have to spend an infinite amount of energy on its acceleration! Imagine the angle  $\phi$  in the above diagram approaching  $90^\circ$  ever more closely and consider how the total energy of the particle increases as it does so!

Reversing our logic we can also conclude that photons do not have a rest mass, since they always have the velocity  $c$ . From  $E_0 = 0$  it follows that  $E_{tot} = p \cdot c$  from (2). These light particles carry not only energy, but also a well-defined momentum  $p = E / c$  within themselves. This momentum of the light particles produces a certain pressure on an illuminated surface. In addition, there is a particularly beautiful illustration of the effect: Astronomers have known for a long time that sunlight exerts a radiation pressure on the tail of a comet. The comet tail always flows away from the sun. As a comet moves away from the sun, it does not pull its tail behind it, but instead the tail flies ahead of it! The dust tail, which consists of heavier particles, appears somewhat more lethargic than the gas or ion tail, which consists mainly of water molecules. The following picture beautifully shows the two components of the tail. It is of the comet Hale-Bopp, as seen in March 1997.



<http://astronomy.swin.edu.au/sao/imagegallery/Hale-Bopp.jpg>

One should not think that the total energy of an object always increases as it moves faster. The important thing is whether energy is input to it or not. Thus the total energy and mass of a streetcar which pulls energy from a power line actually does increase as shown in the Epstein diagram above. However, the situation is different for the battery-operated electric vehicle. It takes the energy needed for its acceleration 'from its own substance', and thus converts electro-chemical energy into kinetic energy. Neither its total energy nor its mass increases. Try drawing the appropriate Epstein diagram!

Algebra delivers further relationships between the energies, momentum and relative velocity. We present the most important here:

$$\cos(\phi) = \frac{v}{c} = \frac{m_0}{m_v} = \frac{m_0 \cdot c^2}{m_v \cdot c^2} = \frac{E_0}{E_{\text{tot}}}$$

$$\sin(\phi) = \frac{p}{m_v \cdot c} = \frac{p}{m_0 \cdot c} = \frac{p}{m_0 \cdot c} = \frac{p}{m_0 \cdot c} = \frac{p}{m_0 \cdot c} = \sqrt{1 - \frac{m_0^2}{m_v^2}} = \sqrt{1 - \frac{E_0^2}{E_{\text{tot}}^2}} = \sqrt{1 - \frac{1}{1 + \frac{E_{\text{kin}}^2}{E_{\text{tot}}^2}}}$$

$$m_0^2 = m_v^2 - \frac{p^2}{c^2}$$

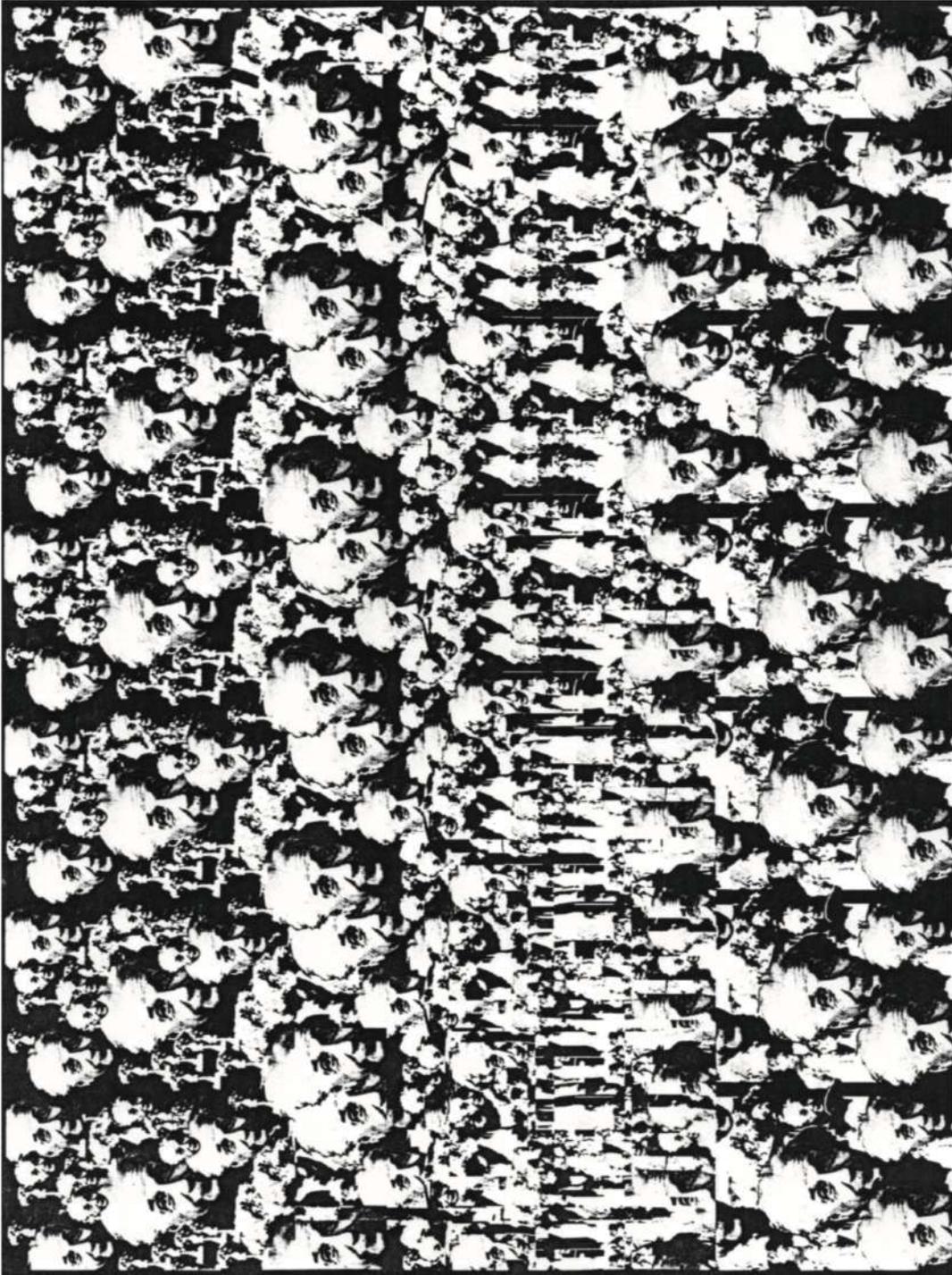
$$E_0^2 = E_{\text{tot}}^2 - p^2 \cdot c^2$$

$$p^2 = (m_v^2 - m_0^2) \cdot c^2$$

Because of its great importance the equation  $\Delta E = \Delta m \cdot c^2$  is always being derived in new ways. Perhaps the most beautiful and simplest derivation is from Einstein himself in 1946 (!) in his very readable book "Out of My Later Years" [22-121ff]. The derivation is not completely accurate (it uses some approximations), however it requires nearly no mathematics and makes few assumptions. It is warmly recommended to the reader.

## E6 Problems and Suggestions

1. How much mass does a radio station radiate daily, if it broadcasts around the clock with an output of 12 kW?
2. How much mass is supplied daily to the earth through its exposure to the sun? Assume a 'solar constant' of  $1400 \text{ W/m}^2$ . What pressure does this radiation exert on the earth ( $\rightarrow$  E5)?
3. In 2005 the total energy consumption of Switzerland according to the Federal Bureau of Statistics amounted to  $890'440 \text{ TJ}$ . How many  $\text{m}^3$  of granite does this correspond to?
4. Calculate in general the  $v/c$  for electrons, which have passed through a given accelerating voltage  $U$  a) classically and b) relativistically
5. For a few years now HighCap condensers have offered capacities of several Farads. However they can not be used with high voltages. A HighCap condenser of 4.7 Farad has a mass of 4 gram when uncharged. What mass does it have, after it is subjected to 12 volts?
6. Chemists always suppose the conservation of mass. Is it not possible however that so much energy is set free with violent reactions that a small mass deficit becomes measurable? Examine the following gas reaction (called an oxyhydrogen reaction): 2 mol  $\text{H}_2$  and 1 mol  $\text{O}_2$  yield 2 mol of  $\text{H}_2\text{O}$ , and thereby an energy of  $2 \cdot 240 \text{ kJ}$  is released. What % of the original mass 'disappears' in this reaction?
7. Which velocity (as % of  $c$ ) results in  $m_v = 3 \cdot m_0$ ? Solve the problem with both a diagram and a calculation!
8. The ratio  $m_v / m_0$  can be taken as a measure for velocities reached in a particle accelerator. Another measure is the difference to the speed of light, and yet another is the energy input into the particles. At the Super Proton Synchrotron in CERN one can accelerate since 1976 protons in such a manner that  $m_v$  is 427 times as large as  $m_0$ . Compute  $v/c$ , the difference  $c - v$ , as well as the necessary acceleration energy in GeV.
9. Continuation of problem 8: The CERN circular tunnel, in which the protons race around, has a radius of 1200 meters. How strong does the magnetic field have to be, in order to hold the protons with assistance of the Lorentz force on the circular path, given that they only have a rest mass  $m_0$ ? What mass  $m_v$  do they have, given that an effective magnetic field of 1.11 Tesla is required?
10. The energy of the electrical field of a charged sphere amounts to  $q^2 / (2 \cdot 4 \cdot \pi \cdot \epsilon_0 \cdot r)$ . What radius results for the electron if one assumes that its rest mass is nothing more than the mass which corresponds to the energy of its electrical field? (Note: there are no experiments, which prove a spatial expansion of the electron)
11. We know from E1 that momentum is invariant perpendicular to  $v$ :  $p_y' = p_y$ . If one defines force as temporal change in momentum ( $F = dp / dt$ ), then it is easy to show how forces perpendicular to  $v$  are transformed. From this one can derive that the pressure is an invariant. The general gas equation  $p \cdot V = n \cdot R \cdot T$  then provides the transformation of temperature, and the transformation of temperature provides the transformation of energy...



This is a so-called 'autostereogram': Fix your gaze at a point some 40 cm behind the picture, while letting your eyes adapt to the picture itself. Some never succeed while for others the 3-D effect precipitates almost immediately as they 'sink into the picture'. With some effort most eventually succeed and subsequent success usually comes more easily...

By the way: It is *much* more impressive in the landscape format!



Spiral Galaxy NGC 1097  
(VLT MELIPAL + VIMOS)

## F Conservation Laws

In the first section we present the central credo of classical physics on a single page. In the second section we look at the changes that STR undertakes on it. Then examples of the principle conservation laws of physics follow: We look at some important processes, through which mass is converted into energy, and fundamentally consider the relativistic impact on the process. Finally we discuss the processes through which particles are produced from energy or through which particles and antiparticles 'dematerialize' into pure energy.

For the transformation of electrical and magnetic fields we refer to the representations of other authors. These transformations were actually the goal of Einstein's initial work on special relativity theory, because they fix the "asymmetries which do not seem to be inherent in the phenomena".

## F1 The Credo of Physics

The four most important quantities of physics are:

1. **electric charge**, a scalar with symbol  $q$  and unit Coulomb
2. **mass**, a scalar with symbol  $m$  and unit kilogram
3. **momentum**, a vector with symbol  $\mathbf{p}$  and unit  $\text{kg} \cdot \text{m/s}$  or  $\text{N} \cdot \text{s}$
4. **energy**, a scalar with symbol  $E$  and unit Joule.

Why these four and no others? The answer to this question is the central credo of classical physics: In a closed system the total amount of these four quantities remains constant, no matter what else happens! One can neither produce nor destroy electrical charges, one may dissociate or transfer them, but the sum of all positive and negative charges always remains constant. Also the total mass of the scrap heap after a mass collision is equally large as the sum of the masses of the individual cars involved in the collision. But the momentum, will it not be destroyed when I crash to the ground? No, not if one includes everything involved in the impact (all of these **conservation laws** apply only to closed systems!). The conservation of total energy is an insight of the second half of the 19th century. Energy can be neither produced nor destroyed, but only converted into different manifestations.

On a pedestal next to these core quantities stand Newton's laws of motion:

1. The law 'actio = reactio': There are no individual forces, but only reciprocal reactions.
2. The law of inertia: In the absence of a force, the velocity  $\mathbf{v}$  remains constant (including the case  $\mathbf{v} = 0$ ).
3. The force law: The change in momentum equals the acting force:  $\mathbf{F} = d\mathbf{p}/dt$ .

The first law (together with the third) is equivalent to the conservation of total momentum. We nevertheless do not want to omit it, because it brings, with its Latin conciseness, a very deep insight. The second law is a special case of the third; it only stands still in order to annoy Aristotle a little. The third law is however indispensable: It tells us how the future motion of a particle is influenced by the forces acting on it.

Thus we must clarify which forces truly exist. The answer is again easily overlooked. There are only three forces, which originate from three different *vector fields*:

1. The gravitational or Newton force, which acts on mass:  $\mathbf{F}_N = m \cdot \mathbf{g}$
2. The Coulomb force, which acts on electrical charge:  $\mathbf{F}_C = q \cdot \mathbf{E}$
3. The Lorentz force, which acts on fast electrical charges:  $\mathbf{F}_L = q \cdot (\mathbf{v} \times \mathbf{B})$

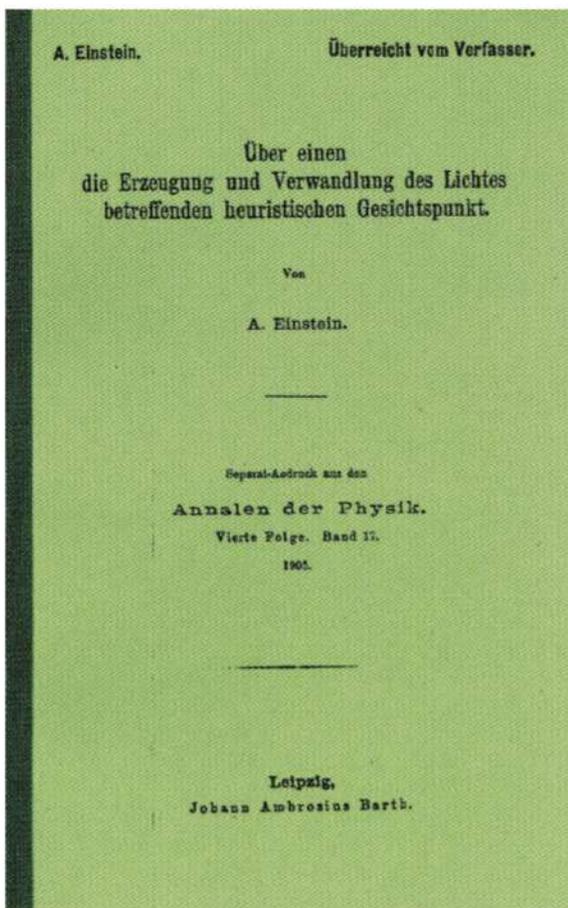
Where, however, do the appropriate fields, i.e., the gravitational field  $\mathbf{g}$ , the electrical field  $\mathbf{E}$  and the magnetic field  $\mathbf{B}$  come from? Newton had already provided the answer for the gravitational field: It not only acts on masses, but it is also produced by the masses. The exact specification gives his gravitation law. Electric and magnetic fields are however produced by electrical charges at rest and in motion. Here the specification is given by the often mentioned four equations of Maxwell. We can only mention and not present in detail these equations, which describe the emergence of the fields.

Thus 5 equations completely describe the origin of the fields, 3 equations describe on what the fields act and the direction of this action and a further equation describes the path of the particle. Together with the 4 conservation laws we have presented the essence of classical physics – all on one page!

It is an enormous mental achievement to ascribe to such a small kernel of basic tenets the richness of the phenomena, which reveals the outside world (its existence is here simply postulated) to an observer. What economy of terminology, what thriftiness of axioms! The geometrical details and the material composition of a gadget may be extremely complicated – yet everything that takes place within it, is completely described by our handful of equations.

Mechanics, thermodynamics and electromagnetism thus cover all phenomena which in the 19th century were considered belonging to physics. It was clear to only a few physicists around 1900, such as Lorentz, Planck and Poincaré that this picture was not as harmonious, complete and self-contained as most thought at the time. The threat did not come from the 'atomists'. The fact that mass is granular and not continuous did not actually disturb anyone. But there was the problem of the movement of the earth through the ether and the thereby expected fluctuations of the speed of light (see **A3**). Max Planck opened a further problem area in 1900: He succeeded in theoretically deriving the experimentally well investigated frequency distribution of radiation from a 'black body' at a given temperature. He had to use however rather adventurous hypotheses concerning the 'granularity' of the radiation energy and also his own unique statistical counting method. Then followed x-rays, the radioactive radium of the Curies, the alpha, beta and gamma radiation of Rutherford and others - nearly every year completely new areas of research were opened. The great building of classical physics was hardly finished and already structural cracks began showing. Various renovations and annexes became necessary.

In the next section we will see an overview of the corrections the STR makes to the core of classical physics, in order to successfully patch one large crack: The incompatibility of Newtonian mechanics, the relativity principle of Galileo and Maxwell's equations.



Also the other large crack, which was opened by Planck's work on radiation, was successfully worked on in 1905 by Einstein. As already mentioned in **A4** he himself called this work "On a Heuristic Point of View Concerning the Production and Transformation of Light" [09-177ff] in a letter to Conrad Habicht 'very revolutionary'. Further work that same spring concerned statistics and provided strong new arguments for the side of the 'atomists'.

## F2 The Relativistic Corrections

What renovations does the SRT make on the building of classical physics, in order to eliminate the internal contradictions between mechanics, the relativity principle and the theory of electromagnetism? It is actually few - yet they are very fundamental:

1. Time measurements are always relative to a coordinate system and not universal
2. Length measurements are always relative to a coordinate system and not universal
3. Inertial mass is (likewise) dependent on relative velocity
4. Input of energy means also an input of inertial mass

The details to the renovations were precisely developed in the preceding chapters. What however are the consequences for the 4 most important quantities of physics and their associated conservation laws?

1. The conservation law of electrical charges goes unchanged
2. The conservation law of inertial mass merges with that of energy to give a single conservation law, in which each energy quantity corresponds to a given inertial mass and vice-versa
3. The conservation law of momentum goes unchanged, whereby the momentum is to be computed as  $m(v) \cdot v$ , thus making the mass dependent on relative velocity

What remains of the three Newtonian laws? Interestingly enough all three remain valid, only the relativistic specification of the momentum must be accounted for. In particular  $\mathbf{F} = d\mathbf{p}/dt$  remains unchanged.

And how is it with the action of forces and the associated force fields? Are there still three? Here, the answer is a "yes, but...". Since the STR confers Maxwell's theory with an unqualified validity in all inertial systems, it is no surprise that Coulomb's law and Lorentz's law remain valid. Also there is not the slightest change in the production of the corresponding fields. The 'but' refers to the production of the gravitational field: The instantaneous action at a distance of masses in Newton's gravitation law contradicts the STR result that  $c$  is a fundamental speed limit for mass, energy and information transfers. By the way, Newton also found this action at a distance to be somewhat uncanny. At the end of his great work he writes:

"Thus far I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity. Indeed, this force arises from some cause that penetrates as far as the centers of the sun and planets ... and whose action is extended everywhere to immense distances, always decreasing as the squares of the distances. ... I have not as yet been able to deduce from phenomena the reason for these properties of gravity, and I do not 'feign' hypotheses. ... And it is enough that gravity really exists and acts according to the laws that we have set forth and is sufficient to explain all the motions of the heavenly bodies and of our sea." [03-943]

Einstein started working in 1906 to integrate gravitation into the STR. He found a point of attack in 1907 with the equivalence principle. He still needed years of hard work and the assistance of some mathematician friends, before he could submit at the end of 1915 the equation, which encompasses space, time and gravitation and solves this problem. He later called the equivalence principle "the happiest thought of my life". More concerning this follows in the next section, which coincidentally has the letter **G** assigned to it.

We still want to turn to the new conservation law, which replaces the separate conservation laws for mass and energy. It can be formulated alternatively as the conservation law for the entire inertial mass in a closed system, whereby all amounts of energy  $\Delta E_i$  with their contribution  $\Delta E_i / c^2$  to the total mass must be taken into account. Or alternatively seen as a conservation law for total energy, whereby all masses  $m_i$  with their contribution  $m_i \cdot c^2$  to the total energy are accounted for. Usually this second representation is preferred. I would like to illustrate the two equivalent possibilities with an example:

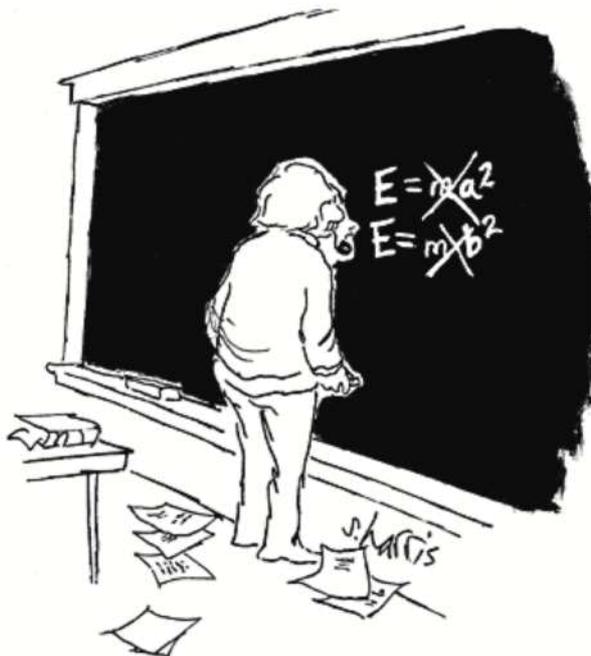
We imagine an uncharged capacitor with rest mass  $m_0$ . What is its contribution to total mass, if it is first charged and then accelerated? During charging the energy  $\Delta E = 0.5 \cdot C \cdot U^2$  is supplied to it, and therefore its rest mass increases by the amount  $\Delta E / c^2$ . This increased mass must still be divided by the root term, when the capacitor is accelerated. This contributes  $(m_0 + \Delta E / c^2) / \gamma$  to the total mass.

The contribution to the total energy is computed as follows: There is the rest energy  $m_0 \cdot c^2$ , and then the energy  $\Delta E = 0.5 \cdot C \cdot U^2$ , supplied by charging of the capacitor at rest, and finally the kinetic energy due to the acceleration. However the acceleration is performed on the already somewhat heavier *charged* capacitor and therefore we must use  $(m_0 + \Delta E / c^2) \cdot c^2 \cdot (\gamma - 1)$  for the kinetic energy. In total we have  $m_0 \cdot c^2 + \Delta E + (m_0 + \Delta E / c^2) \cdot c^2 \cdot (\gamma - 1) = (m_0 + \Delta E / c^2) \cdot c^2 \cdot \gamma$ , which corresponds exactly to the total mass multiplied by the factor  $c^2$ !

It is rather arbitrary, but not wrong, if one still calls this comprehensive conservation law the 'conservation law of total energy'. The designation 'conservation law of total mass' would be just as correct. The two balances differ only by a factor  $c^2$  on each side of the equals sign:

$$\sum E_{\text{tot},i} (\text{before}) = \sum E_{\text{tot},j} (\text{after}) \quad \text{or} \quad \sum m_{v,i} (\text{before}) = \sum m_{v,j} (\text{after})$$

So much for physics from the eagle perspective. The following sections give examples to these remaining three conservation laws. They also show that the world cannot be understood without STR.



Caricature by Sidney Harris  
©ScienceCartoonsPlus.com

## F3 Examples of Mass-Energy Conservation

### The Fusion of Hydrogen into Helium

If a cloud of hydrogen gas is sufficiently hot and dense, then there will no longer be H<sub>2</sub> molecules, nor even H atoms, but only a plasma of free protons and electrons. In this plasma direct collisions of protons occur very frequently. At lower temperatures they can be slowed to zero by the coulomb repulsion (there is always a coordinate system, in which the total momentum of two particles is zero...) and then propelled back to where they came from. If the collision is not completely direct, then they whiz past each other on hyperbolic orbits. If the temperature is however sufficiently high, then they impact in a direct collision in such a manner that the short ranged strong force between the two nuclear particles begins to act and unites them as a deuterium nucleus consisting of a proton and a neutron. This reaction emits a positron e<sup>+</sup> and a neutrino ν. The neutrino is needed only to satisfy further conservation laws (here the lepton number) of particle physics. The positron will soon encounter an electron e<sup>-</sup>, whereby the two particles 'annihilate' each other, i.e. dematerialize into two energy quanta (aka photons) (see F5).

Two deuterium nuclei could then fuse directly into a He nucleus, consisting of two protons and two neutrons. More frequently however a further proton will merge with the deuterium into a He-3 nucleus and two such He-3 nuclei will fuse to a normal He-4 nucleus while emitting two protons. Yet other fusions are possible – but in the long run it is always the case that from 4 protons and 2 electrons a He-4 nucleus is created while emitting two neutrinos.

Nun kennt man die Ruhemassen all dieser Teilchen mit hoher Präzision (Stichwort Massenspektrograph). Wir stellen eine Massenbilanz auf:

before	4 protons	4·1.007,825 u	
	2 electrons	2·0.000,056 u	
	total		4.03,420 u
after	1 He-4 nucleus	4.002,603 u	
	2 neutrinos	2·0.000,000 u	
	total		4.002,603 u
'missing' mass			0.029,817 u

The fusion of only one He nucleus from protons frees an amount of energy corresponding to 0.029,817 atomic mass units. If we fuse a whole mol of helium, then we can multiply this amount by Avogadro's number and obtain about  $2.6 \cdot 10^{12}$  J. With this fusion  $0,029,817 / 4.032,420 \approx 0.74\%$  of the original mass 'disappears'.

This fusion process occurs as we said only under extreme conditions (hydrogen bombs must therefore be ignited with an 'ordinary' uranium bomb...). No material container could include such a plasma. Research reactors are currently being built however, in which this process can be run in a controlled environment. Fusion would have the tremendous advantage over nuclear fission that it does not produce long-lived radioactive substances.

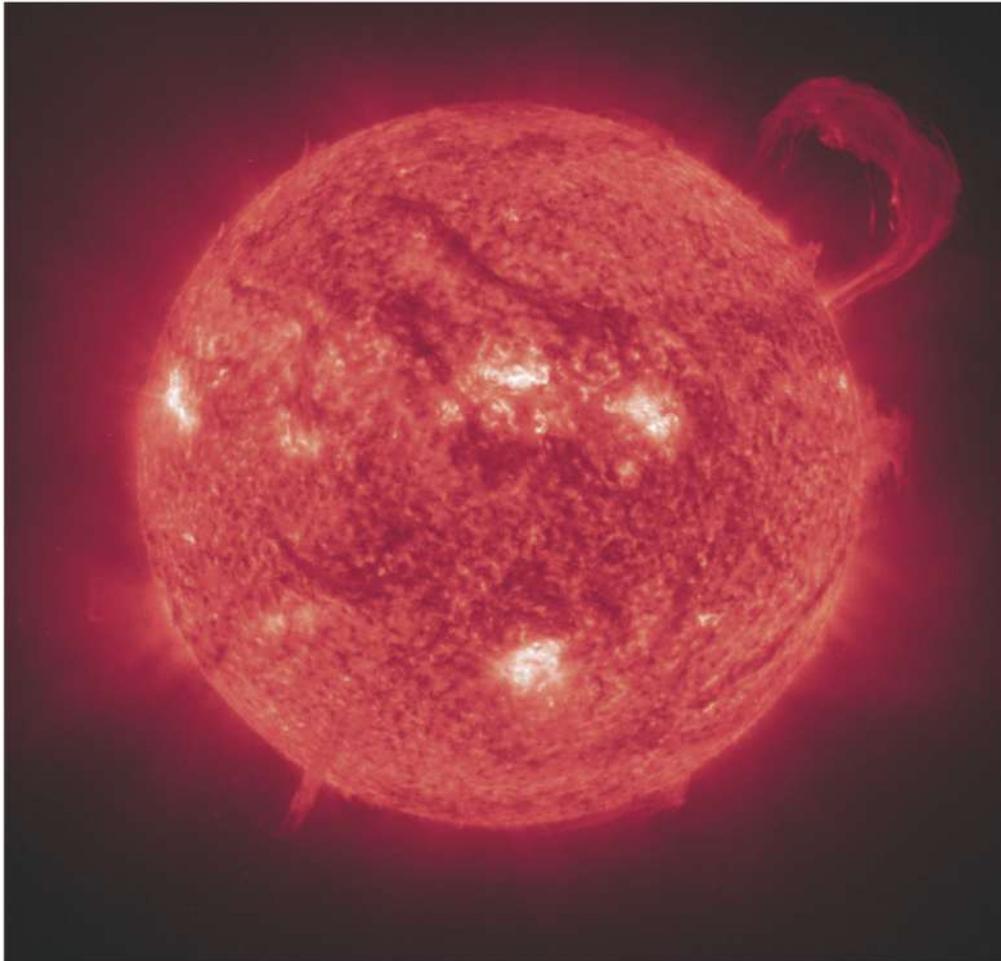
By the way the fusion reaction we described also illustrates the conservation of electrical charge!

### The Squandering of Energy by Our Sun

The sun has been radiating tremendous amounts of energy for millions of years. Still around 1900 one did not have the slightest idea where it got its energy. One could figure out that a sun made from pure coal (considered apart from the oxygen needed to burn) would burn out after a few 1000 years. Today one would naturally do the calculation with oil...

The total energy output of the sun can be computed quite simply: In the alps one measures an energy flow of approximately  $1380 \text{ W/m}^2$ , the so-called 'solar constant'. If one assumes that the sun delivers its radiation symmetrically in all directions of a sphere, then one can multiply this value by the surface area, whose radius is the average radius of the Earth's orbit. In this way one obtains the  $3.85 \cdot 10^{26} \text{ W}$ , with which to label the light bulb 'sun'.

This energy is essentially produced (as with all 'main sequence stars') by the fusion of hydrogen into helium.  $3.85 \cdot 10^{26} \text{ Js}$  per second are radiated away. We obtain the appropriate mass loss, if we divide this number by  $c^2$ : Per second the sun loses about  $4.28 \cdot 10^9 \text{ kg}$  of matter - that is 4.28 million tons! In one year that amounts to  $1.35 \cdot 10^{17} \text{ kg}$ , and in 10 billion years  $1.35 \cdot 10^{27} \text{ kg}$ . Considering this value in relationship to the total mass of the sun:  $1.35 \cdot 10^{27} / 1.99 \cdot 10^{30} \approx 0.000,678$ , we see that in 10 billion years the sun loses less than 1 part per thousand of its entire mass!



SOHO - Picture of the sun on September 14th, 1999 during an enormous eruption, which shows up intensively as UV light of ionized helium at 304 angstroms <http://soho.esac.esa.int/gallery/images/superprom.html> (© ESA and NASA)

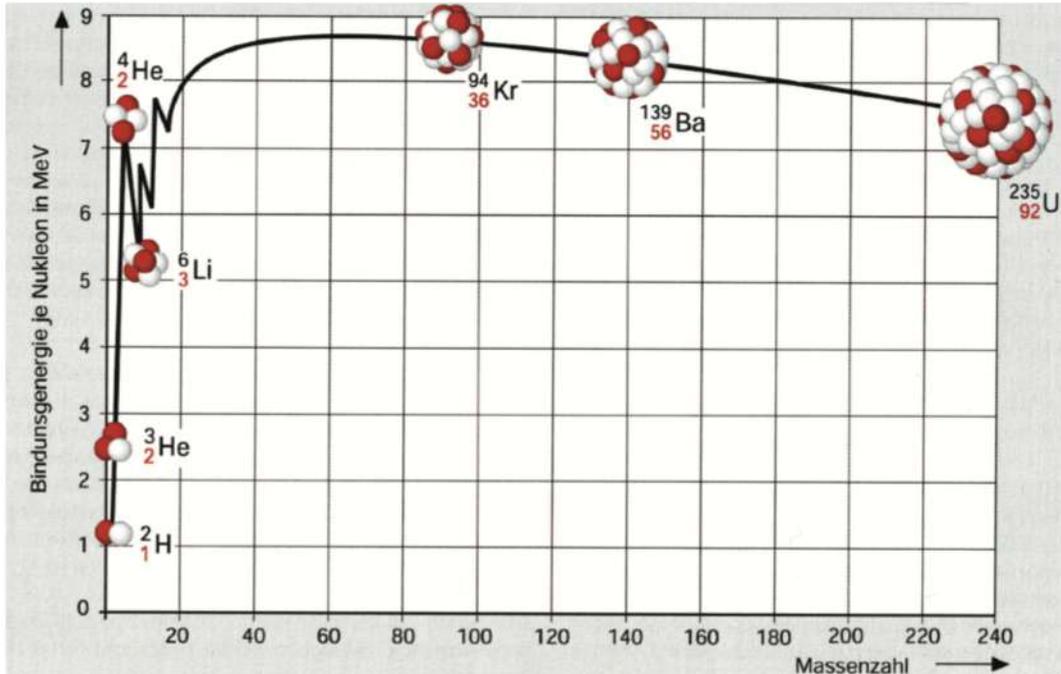
Even though the sun radiates less than 1 part per thousand of its mass in 10 billion years, it nevertheless reaches the end of its time as a main sequence star, because in its center, where the extreme conditions necessary for fusion prevail, the concentration of hydrogen decreases strongly compared to that of helium. As every second  $4.28 \cdot 10^9$  kg of matter are radiated away, then that is the 0.74 % of the hydrogen mass which does not appear any more in the helium. Therefore every second  $5.78 \cdot 10^{11}$  kg hydrogen must be converted into helium. Thus the respective changes in hydrogen and helium concentrations can be computed yielding a model of the sun, which gives the pressure, temperature and chemical composition depending on the distance from the sun's center and the age of the sun. In a state of equilibrium the pressure produced by the radiation at each distance  $r$  from the center must offset the gravitational pressure of the outside layer.

Today we assume that the sun and the planet system formed approximately 5 billion years ago from the 'waste' of an earlier star generation (otherwise there would be no heavy elements such as carbon, oxygen, iron and uranium on earth). The sun will continue radiating quite stably and with the same intensity for about 5 billion years. Then another phase will begin...

Today astrophysics can model in great detail the birth, life and death of different types of stars. Here I hoped only to give you a little taste.

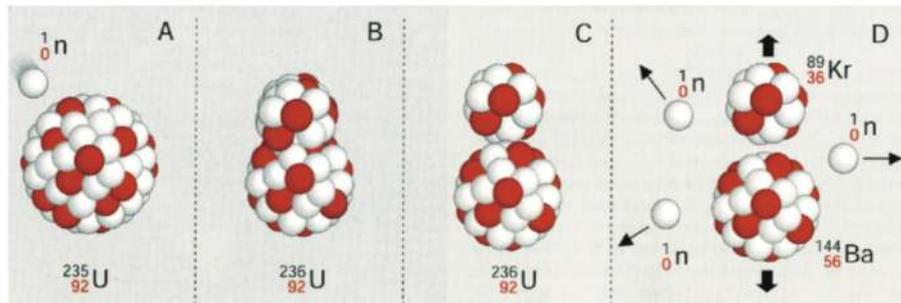
### Radioactive Decay and the Splitting of Heavy Atomic Nuclei

The two protons and the two neutrons in helium are held together by the strong force. This binding energy corresponds to the energy that is released during fusion. It is now possible to determine the middle binding energy per nuclear particle for all atoms, or even better, for all isotopes giving the following diagram:



The energy gain is particularly large by the fusion of protons to helium. In addition, one can win energy, if one splits heavy nuclei. In nuclei that are heavier than iron (Fe-57), the nuclear particles are again on the average less strongly bounded together. Thus energy is released, if one splits one heavy nucleus into two moderately heavy nuclei.

In particular the uranium isotope U-235 needs only to be bombarded with neutrons of suitable kinetic energy, in order to elicit its decay into Kr-89 and Ba-144, for example. In the process three other fast neutrons are produced, making for a suitable nuclear chain reaction:

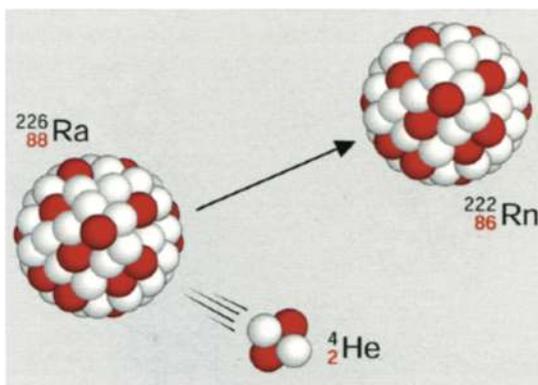


If we know the rest masses of the nuclei involved, we can once again do a balance sheet as we did with the fusion of hydrogen to helium. The two fission products are however extremely unstable (by far too many neutrons in the nucleus) and therefore their rest masses are not specified in the tables. Thus we make another calculation (which in the long run likewise depends on the precise measurements of the rest masses): In Uranium-235 the middle binding energy per nucleus is about 7.6 MeV, with Krypton-89 this amounts to the appropriate value of 8.6 MeV and with the Barium-144 it is about 8.4 MeV (see table above). The result is that the splitting of a single U-235 nucleus releases an energy of

$$89 \cdot 8.6 \text{ MeV} + 144 \cdot 8.4 \text{ MeV} - 235 \cdot 7.6 \text{ MeV} \approx 198 \text{ MeV}$$

Since both products practically immediately decay further (beta decay), there are some additional MeV set free, with which one comes to a total energy of 210 MeV for splitting one U-235 nucleus. Let's extrapolate for a mol of Uranium-235: The complete splitting of 235 grams of U-235 releases the energy of  $6.02 \cdot 10^{23} \cdot 210 \text{ MeV} \approx 2.0 \cdot 10^{13} \text{ Joules} \approx 20 \text{ TJ}$ . The corresponding 'mass loss' is  $20 \text{ TJ} / c^2 \approx 0.225 \text{ gram}$  or somewhat less than one part per thousand.

Excellent information about the fundamentals of fission technology and the different reactor types in use is offered in the publication [23], which was published by the German nuclear power station operators and also from which the three illustrations in this section were taken. Only the section "Disposal of Highly Radioactive Wastes" is - given the present state of the project work - rather tersely handled ...



Energy is also released during radioactive decay: The helium nucleus spontaneously released during  $\alpha$ -decay of a radium atom contains a lot of kinetic energy. This topic is also excellently covered by [23]. I would like to thank the Vattenfall Europe AG and the Informationskreis KernEnergie in Berlin for permission to reproduce the three illustrations shown in this section.

## F4 Relativistic Collisions

### 1. Inelastic Frontal Collision of Two Like Particles, One at Rest

A particle of rest mass  $m_0$  collides with  $v = 12/13 \cdot c$  with a particle at rest of the same type and thereby merges into a new particle of rest mass  $M_0$ , which after its creation has a velocity  $u$ . The conservation of momentum and the conservation of energy-mass must be met:

$$m_v \cdot v + m_0 \cdot 0 = M_u \cdot u \quad \text{and} \quad m_v \cdot c^2 + m_0 \cdot c^2 = M_u \cdot c^2$$

thus  $m_v \cdot v = M_u \cdot u$  and  $m_0 + m_v = M_u$  together with  $m_v = m_0 / \sqrt{1 - (v/c)^2} = m_0 / (5/13) = 2.6 \cdot m_0$

hence  $u = v \cdot (m_v / M_u) = v \cdot (m_v / (m_0 + m_v)) = v \cdot (1 / (\sqrt{1 - (v/c)^2} + 1)) = 12/13 \cdot c \cdot (1 / (5/13 + 1)) = 2/3 \cdot c$

and  $M_0 = M_u \cdot \sqrt{1 - (2/3)^2} \approx (m_0 + m_v) \cdot 0.745 \approx m_0 \cdot 3.6 \cdot 0.745 \approx 2.68 \cdot m_0$

### 2. Inelastic Frontal Collision of Two Like Particles, Moving in Opposite Directions

Two particles of rest mass  $m_0$  collide head on with  $v = \pm c \cdot 12/13$  and merge to form a new particle of rest mass  $M_0$ . We again write the two conservation laws:

$$m_v \cdot v + m_v \cdot (-v) = M_u \cdot u \quad \text{and} \quad m_v \cdot c^2 + m_v \cdot c^2 = M_u \cdot c^2$$

thus  $0 = M_u \cdot u$  and  $2 \cdot m_v = M_u$  together with  $m_v = m_0 / \sqrt{1 - (v/c)^2} = m_0 / (5/13) = 2.6 \cdot m_0$

hence  $u = 0$  and  $M_u = M_0 = 2 \cdot m_v = 2 \cdot 2.6 \cdot m_0 = 5.2 \cdot m_0$

The numerical difference of the two is not very impressive. This is only because we have in our example not 'approached  $c$ '. For  $v \rightarrow c$ , however, the expression  $v \cdot (1 / (\sqrt{1 - (v/c)^2} + 1))$  for  $u$  approaches more and more  $v$ , which means that the particle produced will also have a speed very close to  $c$ , and therefore  $M_v$  will be *much* larger than  $M_0$ ! Today's particle accelerators deliver speeds that are only a few m/s or even cm/s smaller than  $c$ ! Thus the first method above requires a lot more energy to produce a heavier (possibly hypothetical) particle of a given rest mass, because a larger portion of the input energy is spent on the unavoidable kinetic energy of the particle produced. Only the second method can use the complete input energy to produce the new particle (see problems 4 and 5 in F7).

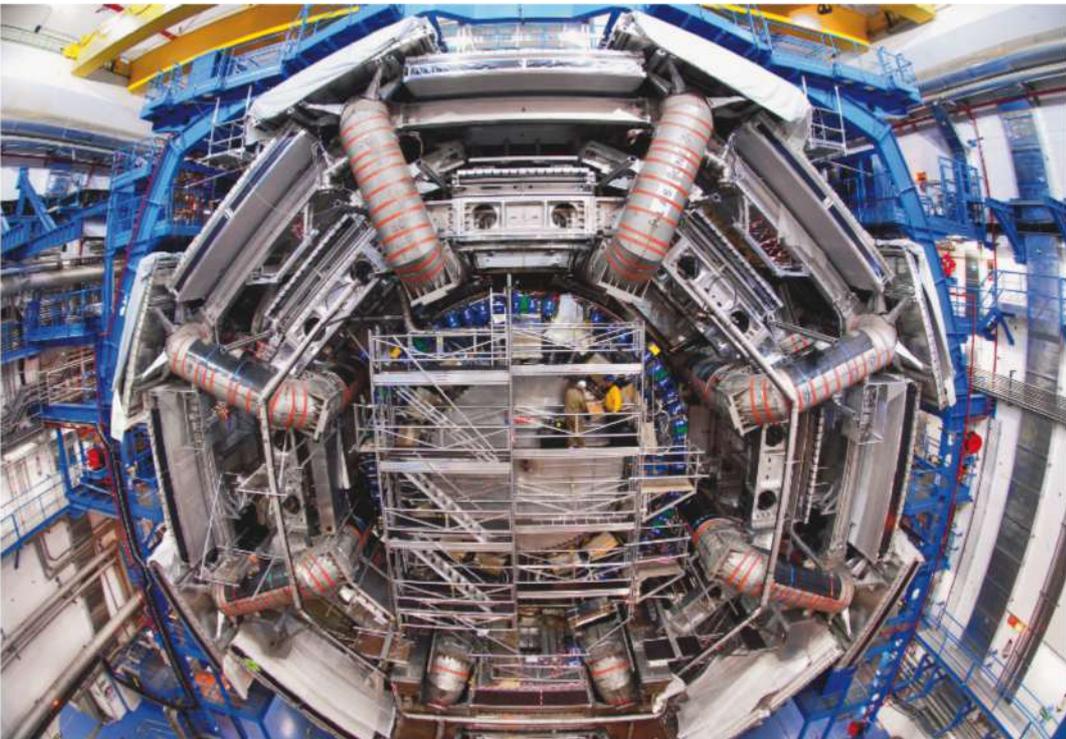
This is the reason that modern facilities like to be equipped with double storage rings, in which the particles (or particles and anti-particles) race around in opposite directions at speeds close to  $c$ , before being brought to frontal collisions inside huge detectors. Such a facility in the vicinity of Hamburg (DESY ~ German Electron Synchrotron) for electrons and positrons has already been active for many years. See the relevant section in the book [11] by Sexl! CERN near Geneva is at the moment (2006) expanding its large plant expressly to handle much heavier protons (LHC ~Large Hadron Collider).

Both the DESY ([www.desy.de](http://www.desy.de)) and CERN ([www.cern.ch](http://www.cern.ch)) provide informative websites. It was, incidentally, at CERN that Tim Berners Lee developed the Internet in its present form in order to facilitate teams whose members live and work in various corners of the world.



View of the 28 km circular tunnel which lies 100 m below the earth's surface. In the spring of 2005, superconducting magnets were installed to hold the protons in their circular path through the two storage rings

[http://doc.cern.ch/archive/electronic/cern/others/PHO/photo-ac/0504028\\_06.jpg](http://doc.cern.ch/archive/electronic/cern/others/PHO/photo-ac/0504028_06.jpg) (© CERN)

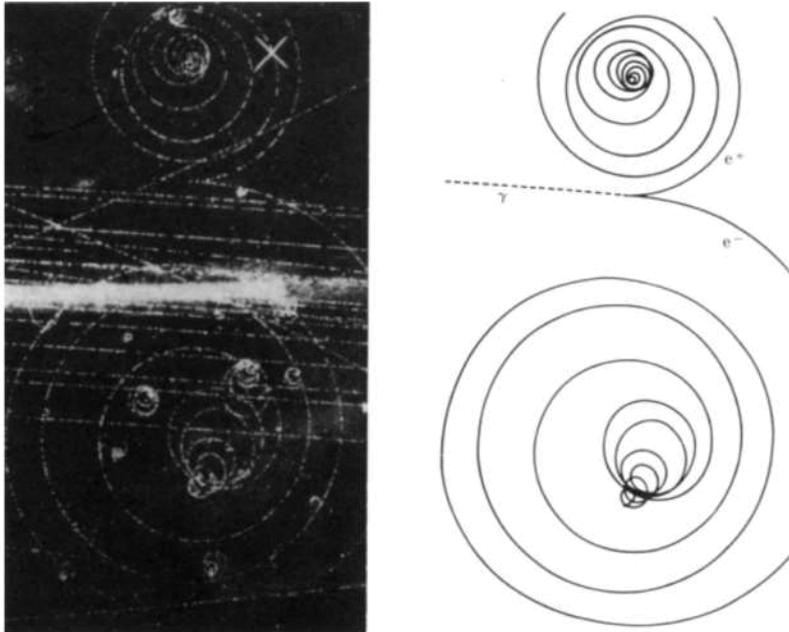


ATLAS, one of the four enormous detectors that record the results of the direct collision of the protons. It gathers in a very short time an amount of data equivalent to the entire European telecommunications network.

[http://doc.cern.ch/archive/electronic/cern/others/PHO/photo-ex/0611040\\_02.jpg](http://doc.cern.ch/archive/electronic/cern/others/PHO/photo-ex/0611040_02.jpg) (© CERN)

## F5 Creation and Annihilation of Particles

The picture below shows the emergence of an electron-positron pair from a photon of high energy, a so-called  $\gamma$ -quant. The photon does not leave a trail in the bubble chamber, because it has no charge. The electron and the positron, through use of a magnetic field perpendicular to the plane of the image, are diverted by the Lorentz force in opposite directions because of their different electric charge. The photon must have entered the picture from the left. We see all three conservation laws in action at once:



The momentum (and hence also the total energy, kinetic energy and velocity) of the electron and the positron can be determined from the radii at the beginning of the spiral motion, since the strength of the applied magnetic field is known. It is

$$m_e v^2/r = e \cdot v \cdot B \quad \text{thus} \quad p = e \cdot r \cdot B \quad \text{and further} \quad E_{\text{tot}}^2 = E_0^2 + p^2 \cdot c^2$$

Problem 7 in Section F7 refers to this situation.

Pions, muons and other particles are continuously generated by the millions through impact of high energy cosmic radiation with the atoms of the Earth's atmosphere. Anti-protons are now manufactured in large numbers at CERN and also at CERN scientists have generated anti-hydrogen atoms from anti-protons and positrons.

When a positron meets an electron the two particles 'decay' into two photons. The conservation of momentum requires that at least two photons be produced: the total momentum of a single photon cannot be zero in any coordinate system, while the total momentum of two particles in their center-of-mass (barycentric) system is always zero! For the same reason, a photon cannot generate an electron-positron pair without another particle being involved. The event always takes place in the immediate vicinity of an atomic nucleus.

Surf-Tip:

[http://teachers.web.cern.ch/teachers/archiv/HST2005/bubble\\_chambers/BCwebsite/index.htm](http://teachers.web.cern.ch/teachers/archiv/HST2005/bubble_chambers/BCwebsite/index.htm)

## F6 How to Continue?

If we just want to show the main results of the STR, then the next chapter should continue as follows:

- A preliminary section, in which we work out how to transform forces and accelerations. We have provided all of the prerequisites.
- A second section, in which we examine what happens to the Lorentz force, which acts on moving electrons, when represented in the system of the moving electrons itself.
- Then we should in general derive how electric and magnetic fields transform in the STR.

These three points are essential. Only then can we achieve the goal of Einstein - to explain the "asymmetries which do not seem to be inherent in the phenomena". Perhaps in time an expanded edition of this book will appear – but for the moment, I would simply like to reference the following works of other authors:

- Michael Fowler, in his Internet accessible script [24], gives a basic introduction to the frame dependence of the electromagnetic field. [24] is in general a very nice elementary presentation of the STR and the only other one that I found which also quantitatively presents de-synchronization as a basis phenomenon!
- Roman Sexl and Herbert K. Schmidt present in chapter 16 of [25] a derivation of the transformation of the electromagnetic quantities without the use of higher mathematics. They employ four component vectors in their calculations. This elegant mathematical representation of the STR is introduced in an easily comprehensible fashion.
- Jürgen Freund's book [26] presents in Part IV an introduction to computing with four component vectors. Using this he then derives the transformation of the electric and magnetic field in the STR in a way similar to Sexl et al. [25].
- Anyone with an elementary knowledge in matrix calculations may enjoy a visit to the section "Maxwell" of "www.relativity.li" !

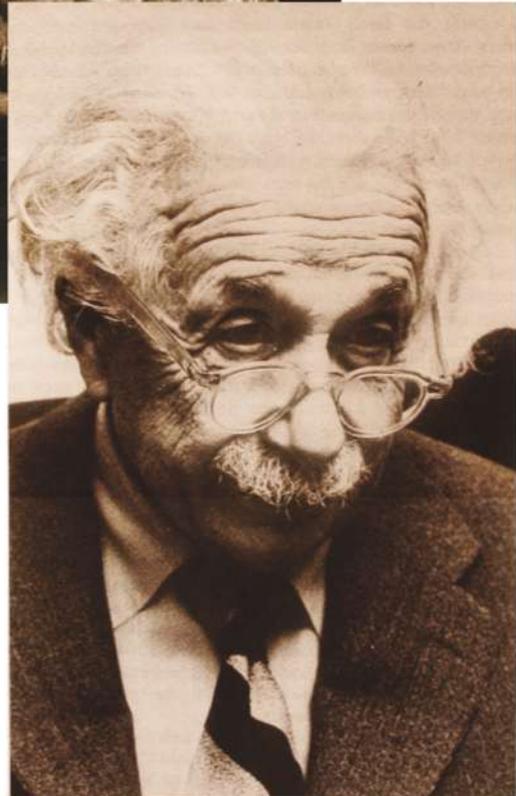
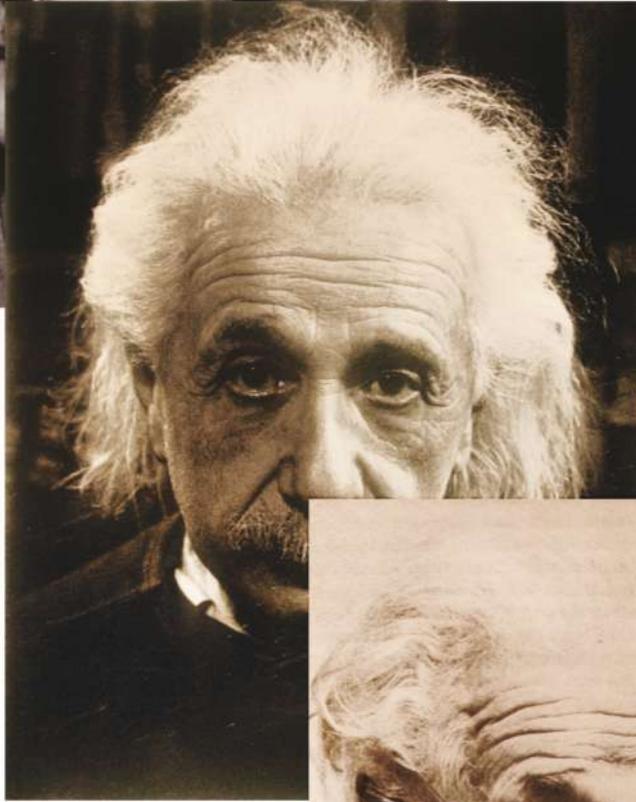
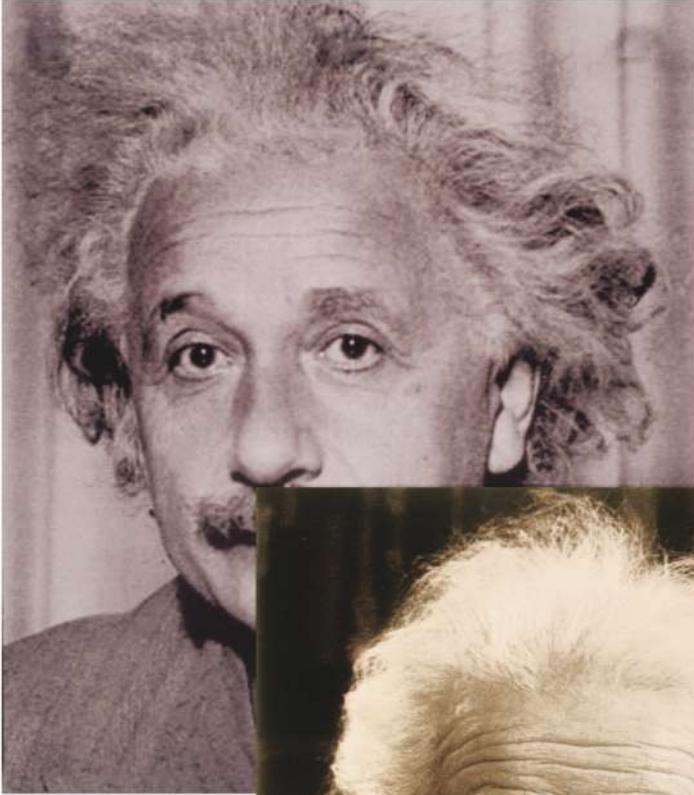
Some other topics we have treated could also be investigated in more detail:

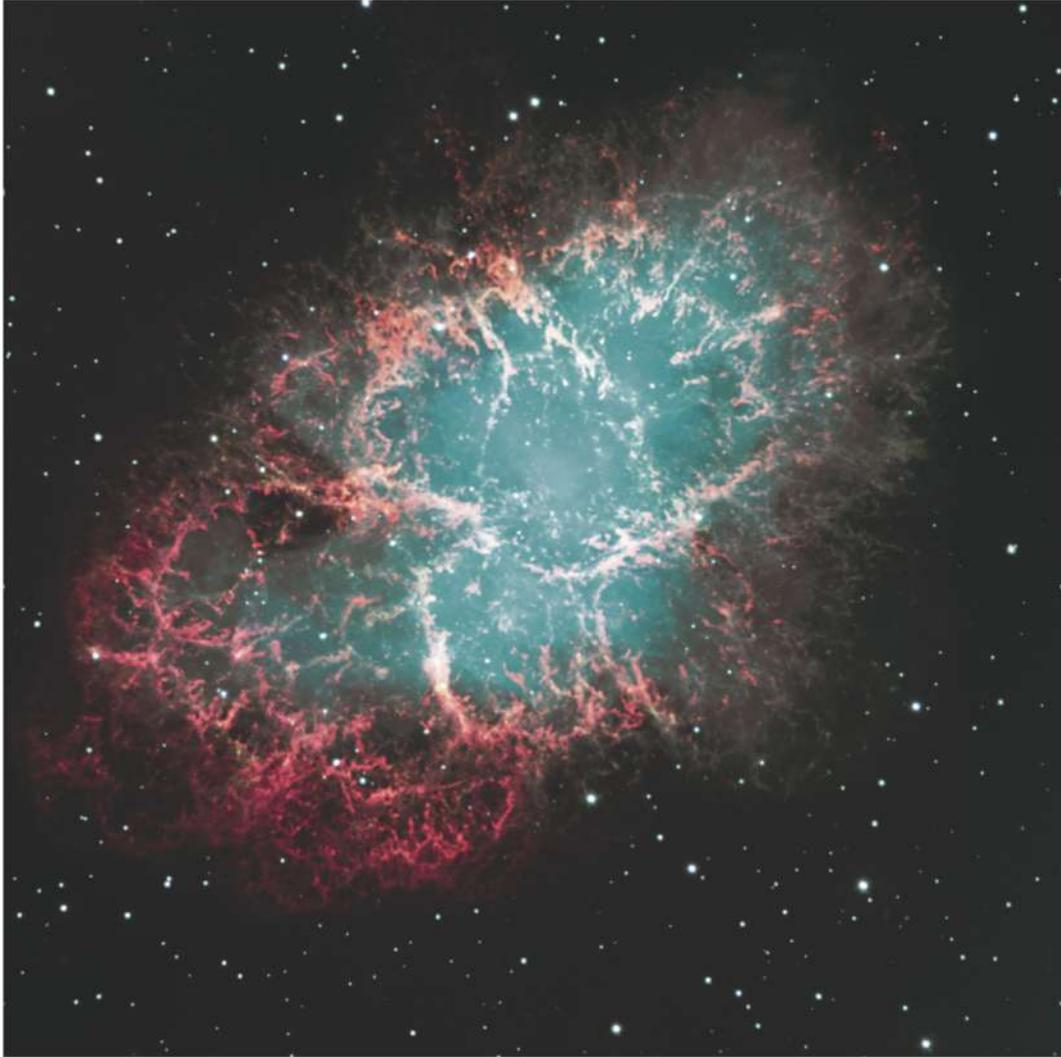
- Transformation and the addition of arbitrary speeds. We have only considered velocities parallel and perpendicular to  $v$ . One could also derive the general formula for aberration.
- General Doppler formula. We have investigated only frequency changes in motion in the radial direction (the 'longitudinal' Doppler effect)
- Transformation of the quantities of thermodynamics. Some hints to this are given in problem 11 of **E6**.

Several additions and suggestions will be presented in chapter **K**. But for now I would like to maintain our momentum and leap into an introduction of the general theory of relativity. I will continue to make use of Epstein's presentation in [15], as well as the beautiful, but long out of print book [27] by Horst Melcher, published in 1968 in the former GDR, and which therefore perhaps has not experienced in the 'West' the renown it deserves.

## F7 Problems and Suggestions

1. Think about how the following energy units must be converted into one another:  
a) J    b) MeV    c) u    d) kg
2. A common oxygen atom (ie O-16) weighs 15.994915 u, a hydrogen atom (H-1) weighs 1.0078252 u. Calculate the average binding energy of a proton or neutron in the oxygen nucleus and compare your result with the diagram in **F3**.
3. A particle of rest mass  $m$  collides inelastically with kinetic energy  $4 \cdot m \cdot c^2$  with a resting particle of the same rest mass. Show mathematically that the particles can merge into a single particle and calculate its rest mass.
4. In a direct collision of an electron with a positron psi particles can be created if the electron and the positron have been accelerated to the point where their masses increase to 3700 times their rest mass.
  - a) Determine the required kinetic energy of the electrons in MeV
  - b) Determine the rest mass of the resulting psi particle
  - c) How heavy would the scrap heap be, if two small cars of rest mass 500 kg collide head on with the same speed as the electron and the positron?
5. (challenging follow-up to problem 4) What energy in MeV must a positron have in order that its collision with a resting electron produces a psi particle? Do not consider the velocity, but rather the energy and momentum and use equation (2) in **E5** ! Compare this result with the effort required using a double storage ring as in 4!
6. Show that you do not obtain the relativistic expression for kinetic energy, if you simply substitute the dynamic mass  $mv$  for the variable  $m$  in the formula  $0.5 \cdot m \cdot v^2$ .
7. This problem refers to the illustration in **F5**. The magnetic field perpendicular to the plane of the image has a strength of 0.214 Tesla, and the radii of the two trails at the beginning of their spiral are measured with  $r_1 = 8.31$  cm and  $r_2 = 5.17$  cm respectively
  - a) Calculate for both the electron and the positron the total energy and hence their mass immediately after their emergence
  - b) Determine the kinetic energies and velocities the two particles have after their formation
  - c) Determine the minimum energy of the  $\gamma$ -quant that is produced and using Planck's formula  $E = h \cdot f$  also its minimum frequency
  - d) For photons  $E = p \cdot c$ . Show that only part of the momentum of the quant is given to the two resulting particles and thus another particle must also be at play in this production
8. An x-ray quantum with an energy of 100 keV collides with an electron at rest and is absorbed by it. What speed does the electron take on?
  - a) Solve using the conservation of momentum
  - b) What would the electron velocity be, if the total energy of the photon were converted into kinetic energy of the electron?
  - c) What percentage of the incidental energy is not converted into kinetic energy of the ?  
What happens to this part?





The Crab Nebula in Taurus (VLT KUEYEN + FORS2)

ESO PR Photo 40f/99 ( 17 November 1999 )

© European Southern Observatory

## G From Special to General Theory of Relativity

Starting from the puzzling fact that inertial mass and gravitational mass are not experimentally distinguishable, we encounter Einstein's key insight to the integration of gravity in the relativity theory: the equivalence principle. We then explain for which simple but very important case we will describe the quantitative effects. In the fourth section we study the influence of gravity on 'clocks and yardsticks', just as in **B**, where we studied the influence of velocity on length and time. For relatively weak gravitational fields, we can derive correct formulas. These formulas apply to even strong fields on the outer edge of non-rotating spherical masses, but this we can not prove. Next, we can deduce how velocities are transformed between observers, who are 'immersed to different depths in a gravitational field'. This leads to the realization that for a distant observer, the speed of light in a vacuum is no longer the same everywhere!

## G1 A Strange Experimental Fact

Which is heavier - a kilogram of lead or a kilogram of feathers? [28-155] The answer to this supposed joke is actually not self-evident. Newton was perhaps the first to have wondered about the fact that inertial mass and gravitational mass are experimentally not distinguishable from each other (see the quote in **A1**). In a series of pendulum experiments, he personally investigated the matter and noted that the two quantities were proportional to each other. It is only a question of defining the units for mass and force in order to make this proportionality into an equality.

In section **F1**, we almost casually remarked that we have to deal with, in principle, three different concepts of 'mass':

- Mass as 'inertial mass', which resists a change of momentum
- Mass as 'gravitational or heavy mass', which suffers a force in a gravitational field
- Mass as 'gravitating mass', which itself generates a gravitational field

The fact that the 'field-causing' mass can be equated with the 'field-experiencing' mass has never caused a stir. But that the 'inertial' and 'gravitational or heavy' masses should be identical has no logical basis and was therefore repeatedly tested experimentally. The accuracy that Newton's pendulum experiments achieved was about 1:1000. Probably triggered by thoughts of Ernst Mach, the Hungarian Baron Loránd von Eötvös performed, beginning in 1899, precision experiments on this issue. He improved the accuracy of Newton by many powers of ten. Another significant increase in precision was achieved in 1964 by Robert H. Dicke and his team. Given the fundamental importance of these experiments we present the data of [29-1050ff (!), called 'the phone book' ...] in a small table. I have added the drop tower in Bremen because with this experiment for the first time not just torsion forces are being measured: It directly examines whether all test masses fall with identical acceleration. Since 2005, thanks to new catapult equipment a free fall time of around 9.5 seconds is achieved > <https://www.zarm.uni-bremen.de/en/drop-tower/general-information.html> .

who	when	achieved precision
Newton	um 1680	1 : 10 <sup>3</sup>
Eötvös	1899 - 1922	5 : 10 <sup>9</sup>
Renner	1935	7 : 10 <sup>10</sup>
Dicke et al.	1964	1 : 10 <sup>11</sup>
Braginsky and Panov	1971	1 : 10 <sup>12</sup>
Drop Tower in Bremen	since 1990	1 : 10 <sup>12</sup>

Misner et al. [29] call this fact "the uniqueness of free fall" or "the weak equivalence principle". This experimental fact stands at the beginning of every theory of gravitation. All (small) test bodies fall in the gravitational field of a large body at exactly the same speed, regardless of their composition and mass. Newton could not answer why this is so and he would not speculate ('hypotheses non fingo'). A good theory of gravitation must, however, provide an answer to this question.

Starting in 1906 Einstein worked on this problem - and in 1908 he realized that he could, in his typical way, best provide a solution.



[http://www.einsteinjahr-bremen.de/FallturmBremen\\_Einsteinv1\\_300605.jpg](http://www.einsteinjahr-bremen.de/FallturmBremen_Einsteinv1_300605.jpg)  
(November 2007, the link is no longer active)

## G2 The Equivalence Principle

What does Einstein do, when no logical explanation can be found for an experimental result? He makes that result the starting point for a new theory! He simply elevated the unexplained constancy of the speed of light (with Maxwell as his rear guard) to a basic principle and thereby founded the STR. In a similar manner he tackles gravity - he makes the equivalency of inertial and gravitational mass an axiom.

This equivalence principle of Einstein is so important that we will present various formulations of it (considering [29] we should call it the 'strong equivalence principle'):

1. In principal, it is not possible in a local experiment to determine whether a laboratory is suspended in the gravitational field of a large body causing a gravitational acceleration  $\mathbf{g}$  or whether it is gravitationally free and being subjected to a constant acceleration  $\mathbf{g}$ .
2. There are no local experiments that can distinguish whether a laboratory is free falling in a gravitational field or whether it is resting unaccelerated in gravity-free space.
3. In a homogeneous gravitational field all operations run in exactly the same way as in a uniformly accelerated, but gravity-free reference frame.
4. A small laboratory in a gravitational field, falling freely and not rotating, is an inertial frame in the sense of the STR.
5. The effect of gravity can be locally produced (or reversed) by a suitable acceleration.

In the third formulation, the claim that the experiments should be 'local', that is, they should not stretch out over a 'large' area of space, is replaced by the requirement that the gravitational field should be 'homogeneous', which of course in all cases is valid only in a small area to a very good approximation. The third formulation is so vividly presented by Einstein himself in his popular presentation [30] of the relativity theories that it must have been a conscious reference to the description of phenomena in the ship's belly by Galileo (see quote in **A2**):

"We imagine a large portion of empty space, so far removed from the stars and other appreciable masses, that we have before us approximately the conditions required by the fundamental law of Galilei. It is then possible to choose a Galileian reference-body for this part of space (world), relative to which points at rest remain at rest and points in motion continue permanently in uniform rectilinear motion. As reference-body let us imagine a spacious chest resembling a room with an observer inside who is equipped with apparatus. Gravitation naturally does not exist for this observer. He must fasten himself with strings to the floor, otherwise the slightest impact against the floor will cause him to rise slowly towards the ceiling of the room.

To the middle of the lid of the chest is fixed externally a hook with rope attached, and now a 'being' (what kind of a being is immaterial to us) begins pulling at this with a constant force. The chest together with the observer then begins to move 'upwards' with a uniformly accelerated motion. In course of time their velocity will reach unheard-of values - provided that we are viewing all this from another reference-body which is not being pulled with a rope.

But how does the man in the chest regard the process? The acceleration of the chest will be transmitted to him by the reaction of the floor of the chest. He must therefore take up this pressure by means of his legs. If he does not wish to be laid out full length on the floor, he is then standing in the chest in exactly the same way as anyone stands in a room of a house on our earth. If he releases a body which he previously had in his hand, the acceleration of the chest will no longer be transmitted to this body, and for this reason the body will approach the floor of the chest with an accelerated relative motion. The observer will further convince himself that the acceleration of the body, towards the floor of the chest is always of the same magnitude, whatever kind of body he may happen to use for the experiment.

Relying on his knowledge of the gravitational field (as it was discussed in the preceding section), the man in the chest will thus come to the conclusion that he and the chest are in a gravitational field which is constant with regard to time. Of course he will be puzzled for a moment as to why the chest does not fall in this gravitational field. Just then, however, he discovers the hook in the middle of the lid of the chest and the rope which is attached to it, and he consequently comes to the conclusion that the chest is suspended at rest in the gravitational field.

Should we smile at the man and say that he errs in his conclusion? I do not believe we ought to if we wish to remain consistent; we must rather admit that his mode of grasping the situation violates neither reason nor known mechanical laws. Even though it is being accelerated with respect to the 'Galileian space' first considered, we can nevertheless regard the chest as being at rest. We have thus good grounds for extending the principle of relativity to include bodies of reference which are accelerated with respect to each other, and as a result we have gained a powerful argument for a generalised postulate of relativity.

We must note carefully that the possibility of this mode of interpretation rests on the fundamental property of the gravitational field of giving all bodies the same acceleration, or, what comes to the same thing, on the law of the equality of inertial and gravitational mass. If this natural law did not exist, the man in the accelerated chest would not be able to interpret the behaviour of the bodies around him on the supposition of a gravitational field, and he would not be justified on the grounds of experience in supposing his reference-body to be 'at rest'.

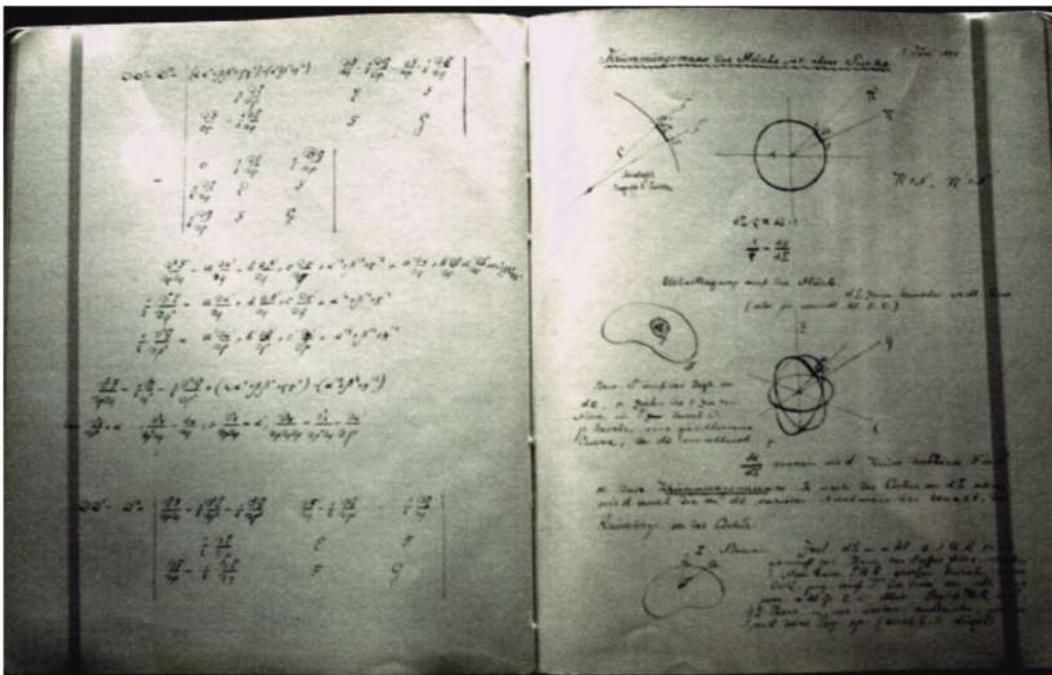
Suppose that the man in the chest fixes a rope to the inner side of the lid, and that he attaches a body to the free end of the rope. The result of this will be to stretch the rope so that it will hang 'vertically' downwards. If we ask for an opinion of the cause of tension in the rope, the man in the chest will say: "The suspended body experiences a downward force in the gravitational field, and this is neutralised by the tension of the rope; what determines the magnitude of the tension of the rope is the gravitational mass of the suspended body." On the other hand, an observer who is poised freely in space will interpret the condition of things thus: "The rope must perforce take part in the accelerated motion of the chest, and it transmits this motion to the body attached to it. The tension of the rope is just large enough to effect the acceleration of the body. That which determines the magnitude of the tension of the rope is the inertial mass of the body." Guided by this example, we see that our extension of the principle of relativity implies the necessity of the law of the equality of inertial and gravitational mass. Thus we have obtained a physical interpretation of this law." [30-75ff]

Maybe this long quotation will incite you to give one of the generally comprehensible texts from Einstein a try. Of course, there are also many drawings, animations, videos and DVDs, which portray these ideas for those not inclined themselves to do the reading. They are all nice and even funny – have a look at your leisure. It is amusing to watch Professor Albert together with his elevator comfortably bolting in free fall down the elevator shaft. I myself am much more interested in the drop tower in Bremen, where one can actually perform such free fall experiments for time intervals up to 9 seconds, than in these humorous drawings meant to illustrate a thought experiment.

### G3 Our Restriction to a Special Case

The (strong) equivalence principle was for Einstein both the starting point and the litmus test for any mathematical formulation of a theory of gravitation. Yet before 1911 he had made little progress. In 1912 he returned to Zurich from Prague to rejoin his friend and colleague Marcel Grossmann, who had meanwhile become a professor at the ETH (Eidgenössische Technische Hochschule = Swiss Federal Institute of Technology). He is reported to have beseeched Grossman: "you must help me or else I'll go crazy." [7-212] Grossmann was quick to help, and Einstein milked mathematics as he had never done before. Soon they found the correct field equations - but rejected them because they thought the first approximations did not agree with Newton's theory. In the summer of 1915 (Einstein had already been in Berlin more than a year) he presented his work to David Hilbert and his people in Göttingen. On November 4th, 1915, he submitted to the Prussian Academy another essay from the series "On the general theory of relativity". A week later he was forced to make a retraction. On November 25th he brought this to an end and published the final version of his equations. Hilbert had already submitted a work on gravitation on November 20th, but it appeared in print only on March 31st, 1916. It also appeared to contain the correct equations. It nearly came to an unattractive controversy. Some still try incorrigibly to create a dispute, but since 1997 we know definitely that the plagiarism accusation can only apply to Hilbert (see [31-105f]).

In a letter to Arnold Sommerfeld Einstein wrote: "Think of my joy ... that the equations correctly predict the precessions of the perihelion of Mercury!" And to his friend Paul Ehrenfest: "I was stunned for several days by joyful excitement." [32-216f, translation by Samuel Edelstein] Einstein was also completely exhausted and had to be looked after for several weeks.



Lecture notes on Differential Geometry by Marcel Grossmann. Einstein frequently played hooky from his mathematics lectures and depended on the (beautiful) notes of his friend to prepare for exams

Einstein writes in his treatise for the Prussian Academy: "The magic of this theory will escape no one who grasps it, it is a true triumph of the methods founded by Gauss, Riemann, Christoffel, Ricci and Levi-Civita in general differential calculus." [32-219, translation by Samuel Edelstein] Einstein's enthusiasm, as well as that of a few other 'insiders', could not really be shared by most people (the author of this book included) since they were not versed in the mathematical 'tools' that were used. Galileo wrote that the book of nature is written in the language of mathematics - but the math of Einstein's theory is for most people an unreasonable demand. A. Herman writes in his very readable biography of Einstein:

"That made some contemporaries angry. The doctor and writer Alfred Döblin said he could understand Copernicus, Kepler and Galileo but the new theory - the 'abominable doctrine of relativity' - excludes him 'and the immense quantity of all people, including the thoughtful and well educated from its insights'. The scientists of today with Einstein at the top had become a 'brotherhood', using 'Masonic signs and communicating in a manner similar to spiritualists with their talking boards'." [32-220, translation by Samuel Edelstein]

As with the STR, in the course of several decades a number of individuals have opened wide the door to a qualitative understanding of the GTR (General Theory of Relativity) for educated non-specialists. We can even perform correct calculations for at least the most important special case, without first needing to complete several semesters of higher mathematics. For me, the two books [15] and [27] already mentioned were most stimulating.



Marcel Grossmann, Albert Einstein, Gustav Geissler and Marcel's brother Eugen during their time as students at the ETH

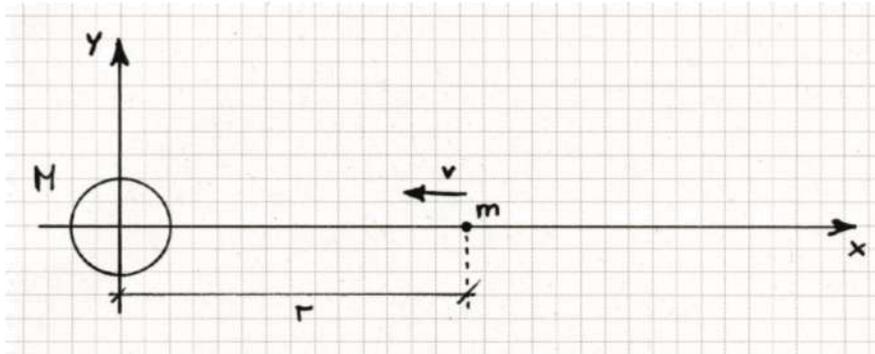
**Our restriction:**

**We only calculate the influence of a single, spherical, non-rotating mass with a rather weak gravitational field on its surface. We treat only the case of a spherically symmetric, weak gravitational field. Weak means: the escape velocity from the surface of the field-producing sphere will be much less than the speed of light.**

Thus for a box, which is in free fall from a great distance toward the surface of the sphere, the classic expression is a very good approximation for the kinetic energy:  $E_{kin} = 0.5 \cdot m_0 \cdot v^2$ . In the diagram in section **E4**, we see that this applies to about  $v = c/3$ . The escape velocity of the Earth is only about 11.2 km/s, and on the sun's surface it is 618 km/s! These values are far below our limit of  $c/3 = 100,000$  km/s, at which point our derivation becomes problematic. Our conditions are extremely well met almost everywhere in the universe except in the vicinity of very exotic objects like neutron stars or black holes. That is particularly true for any location in the gravitational 'catchment area' of our sun.

Strangely enough the formulas we are going to develop within the limits of our restriction and employing very elementary considerations will be the exact result even in the case of a strong gravitational field of a non-rotating massive spherical body!

Now let a small laboratory fall from far away along the x-axis toward the center of a spherical mass M:



Let the small laboratory have the initial speed given by  $0.5 \cdot m \cdot v^2 = E_{kin} = -E_{pot} = G \cdot M \cdot m / r$  at the beginning of the observation. Through conservation of energy this equation is met at each stage of the fall by increasingly smaller values of  $r$  and increasingly larger values of  $v$ . After division by  $m$ , we get

$$v^2 = 2 \cdot G \cdot M / r = -2 \cdot \Phi(r) \quad \text{and} \quad v^2/c^2 = 2 \cdot G \cdot M / (c^2 \cdot r) = -2 \cdot \Phi(r) / c^2 = 2 \cdot \alpha / r = R_s / r$$

with the definitions  $\Phi(r) = -G \cdot M / r$ ,  $\alpha = G \cdot M / c^2$  and  $R_s = 2 \cdot G \cdot M / c^2$ .

$\Phi(r)$  is the classical expression for the potential in a Newtonian gravitational field, and  $R_s$  is the so-called Schwarzschild radius. For our ever-important radical expression, we have

$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{2 \cdot G \cdot M}{c^2 \cdot r}} = \sqrt{1 - \frac{R_s}{r}} = \sqrt{1 - \frac{2\alpha}{r}} = \sqrt{1 + \frac{2 \cdot \Phi(r)}{c^2}}$$

After applying our special preconditions, the value of  $v^2 / c^2$  and thus the value of  $2 \cdot \alpha / r$  becomes very small. The square of  $\alpha / r$  is therefore even much smaller, permitting the following reformulation:

$$\sqrt{1 - \frac{2\alpha}{r}} \approx \sqrt{1 - \frac{2\alpha}{r} + \left(\frac{\alpha}{r}\right)^2} = \sqrt{\left(1 - \frac{\alpha}{r}\right)^2} = 1 - \frac{\alpha}{r}$$

Given the fact that  $\Phi(r) = -(\alpha / r) \cdot c^2$ , we can somewhat simplify our radical:

$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{2\alpha}{r}} \approx \left(1 - \frac{\alpha}{r}\right) = \left(1 + \frac{\Phi(r)}{c^2}\right)$$

We want to once again satisfy ourselves that this approximation is very, very good: The strongest gravitational field in the solar system occurs at the surface of the sun. Check that the value there of  $\alpha / r$  is about  $2.1 \cdot 10^{-6}$  ! We smuggled the square of this expression into our calculation above - something in the range of  $4 \cdot 10^{-12}$  ! This term is one million times smaller than the significant term  $2 \cdot \alpha / r$ .

Our definitions of  $R_s$ ,  $\alpha$  and  $\Phi(r)$  have also earned a red box:

$$\alpha = \frac{G \cdot M}{c^2}$$

$$R_s = \frac{2 \cdot G \cdot M}{c^2} = 2 \cdot \alpha$$

$$\Phi(r) = -\frac{G \cdot M}{r}$$

Now we are prepared to consider the ramifications of the equivalence principle in our important special case and to derive how a gravitational field influences the speed of clocks and the length of yardsticks.

Finally, we want to remind ourselves of the assumptions we have made in the derivation of these formulas. We can formulate these in several different ways:

- speeds resulting from free fall starting from rest should be much smaller than  $c$
- the achievable kinetic energy resulting from free fall starting from rest should be much smaller than the rest energy  $m \cdot c^2$
- potential energy  $m \cdot \Phi(r)$  should always be much smaller than the rest energy  $m \cdot c^2$
- the spherical radius of the central mass  $M$  should be much larger than its Schwarzschild radius, so that the term  $R_s / r$  outside the sphere is everywhere much less than 1

It must also be emphasized that the formulas derived above are only valid in the exterior of the sphere. Inside, the gravitational field loses strength and at its center – from symmetry reasons alone – is zero. This decrease is, according to Newton, linear. We will return to this in **H5**.

## G4 Clocks and Yardsticks in the Schwarzschild Field



Gravitational fields of non rotating spherical masses are often called Schwarzschild fields in honor of the German physicist and astronomer Karl Schwarzschild (1873-1916). Schwarzschild was concerned at the turn of the century with the question of whether the physical space of astronomy is really Euclidean or not. He had already begun in 1913 to look for the redshift of spectral lines of the sun predicted by Einstein. A few weeks after the publication of Einstein's equations, he was the first to present an exact solution, and some weeks later, he delivered a second solution.

He wrote these works while serving in the war on the Eastern Front where he contracted a skin disease from which he died in 1916.

Let us once again consider our little laboratory, as in the last section, starting from the 'OFF' position and falling toward a spherical mass. We want to compare the measurements of an observer in this laboratory to those of a non-moving observer in OFF position, that is, to an observer at rest and at a very large distance from M (and any other large mass).

According to the equivalence principle the freely falling laboratory is at every moment a gravitational-free inertial frame in the sense of the STR. Thus we already know the relationship between the measurements made in the falling laboratory which is at a distance  $r$  from the center of M to those made by the observer in 'OFF' position:  $r$  together with  $\alpha$  (or  $R_s$  or  $\Phi$ ) determine the value of our radical:

$$\Delta t(r) = \Delta t(\infty) \cdot \sqrt{\phantom{x}} = \Delta t(\infty) \cdot \sqrt{(1 - R_s / r)} \approx \Delta t(\infty) \cdot (1 - \alpha / r) = \Delta t(\infty) \cdot (1 + \Phi(r) / c^2)$$

$$\Delta x(r) = \Delta x(\infty) / \sqrt{\phantom{x}} = \Delta x(\infty) / \sqrt{(1 - R_s / r)} \approx \Delta x(\infty) / (1 - \alpha / r) = \Delta x(\infty) / (1 + \Phi(r) / c^2)$$

$$\Delta y(r) = \Delta y(\infty) \quad (\text{no lateral contraction})$$

$$\Delta z(r) = \Delta z(\infty) \quad (\text{no lateral contraction})$$

We obtain these results from the STR and our fourth formulation of the equivalence principle. With free fall we have caused gravitation to completely disappear, replacing it for the observer in the OFF position with a continually adjusted acceleration. The occupants in the free falling laboratory are in fact in an inertial frame the whole time. They must, for example, always measure the same wavelengths in the spectrum of an excited hydrogen atom (the experiment taking place entirely within the falling laboratory).

Assume now they fly past the place  $r$  with the instantaneous velocity  $v = \sqrt{(2 \cdot G \cdot M / r)}$ . In so doing they measure the frequencies in a glowing hydrogen gas cloud which is there at rest in distance  $r$  of M. After taking into account the Doppler effect (the laboratory occupants know their STR) they must obtain the usual well-known values because being in their laboratory system they are indeed in an inertial frame. No such thing as gravitation exists for them!

So the hydrogen gas resting at site  $r$  in the gravitational field radiates at frequencies which are measured correctly with the clocks of our free falling laboratory. But these clocks are slow running as seen from the OFF position! This means, however, that clocks (or any other oscillating systems) at rest at a fixed distance  $r$  from the center of gravity run slower (in comparison with those in the OFF) by exactly the same factor as those in our falling laboratory.

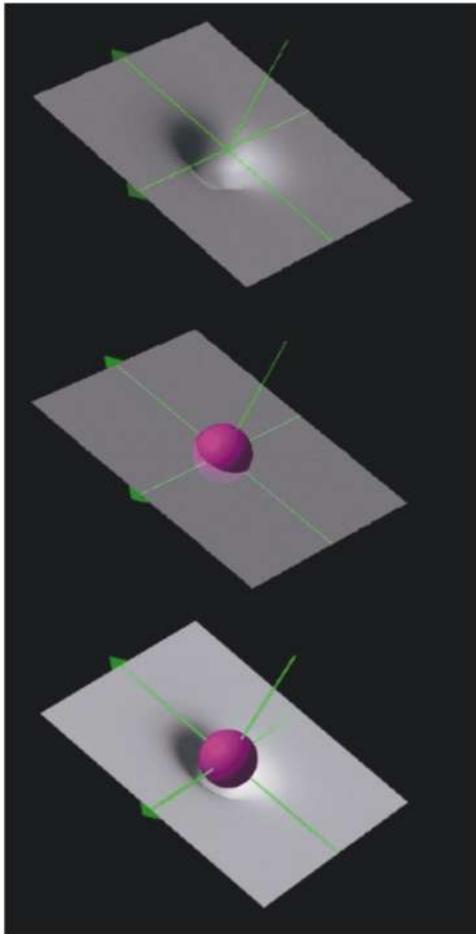
We thus arrive to the following formulation of the equivalence principle, which no longer applies generally, but is instead tailored to our specific situation:

**Measurements of lengths and time intervals, in a small laboratory at rest at place  $r$  in the gravitational field of  $M$  are transformed the same as those of our freely falling laboratory falling past point  $r$  along a field line.**

Considering the above equations and those from the last section, we can derive the following qualitative statements:

- The smaller  $r$  is, the less time elapses when compared to a clock in OFF. The stronger the gravitational field is, the slower the clocks run! Clocks at the same distance from the center of  $M$  run equally fast.
- The smaller  $r$  is, the longer a segment in the radial direction will be when measured with local yardsticks. As seen from OFF: yardsticks shorten in the radial direction with increasing strength of the gravitational field! Thus, for the thickness of a spherical shell around  $M$ , a local surveyor determines a larger value than an observer in OFF.
- The circumference of a circle around the center of  $M$  will be measured equally by local observers and by an observer from OFF.

The second and third points together imply that for an observer in the gravitational field the diameter of a circle is longer than its circumference divided by  $\pi$ ! Therefore the laws of Euclidean geometry no longer apply in a gravitational field. The fact that the diameter is longer than expected (by Euclidean geometry) when measured locally but not when measured from OFF, is usually illustrated as follows:



At the top: A dimple or depression is drawn that has the property that the diameter measured along the gray area is longer than the circumference divided by  $\pi$ . One happily lets a planet circle around this dimple, as if it had a top and bottom and an additional gravitational field in the  $z$ -direction!

Epstein complains vigorously about this “powerfully misleading notion” [15-169]. He is right. With this dimple one is attempting to show only the *metric* in the equatorial plane of the star. The yardsticks always lie in the equatorial plane (middle image), but they shorten in radial direction, when one approaches the surface of the star. In the center of the star they have their minimal length.

To depict this distortion with respect to the Euclidean metric, one extends the equatorial plane into ‘hyperspace’ - whether you have a dimple pointing ‘down’ or a bump pointing ‘up’ is irrelevant (bottom picture). This additional dimension has nothing to do with the  $z$ -direction. The point of inflection of the cross-section of this dimple has the  $z$ -axis distance  $R$ , where  $R$  is the radius of the star as measured from OFF.

For the following, assume we are sitting next to an observer in OFF, that is, very far from the mass M, at a place where the potential  $\Phi(r)$  practically disappears (that is, it approaches zero...). This slightly fictitious position helps when we consider transforming the readings from a laboratory at distance  $r_1$  into those of a laboratory at a distance  $r_2$  from the center of mass. Imagine, for example, a flashing light of constant frequency at point  $r_1$ . What time interval does a local observer at point  $r_2$  measure with his clock until he has counted 100 light flashes?

According to the formulas at the beginning of this section we have

$$\Delta t(r_1) / \sqrt{1 - R_S / r_1} = \Delta t(\infty) = \Delta t(r_2) / \sqrt{1 - R_S / r_2} \quad \text{and thus}$$

$$\Delta t(r_2) = \Delta t(r_1) \cdot \sqrt{(1 - R_S / r_2) / (1 - R_S / r_1)} = \Delta t(r_1) \cdot \sqrt{(1 - R_S / r_2) / (1 - R_S / r_1)}$$

Similarly, for small lengths in the radial direction (i.e., in the x-direction)

$$\Delta x(r_2) \cdot \sqrt{1 - R_S / r_2} = \Delta x(\infty) = \Delta x(r_1) \cdot \sqrt{1 - R_S / r_1} \quad \text{etc.}$$

(Small) segments orthogonal to the field lines are, however, for all observers of equal length.

Using our approximations we get the following simpler results:

$$\Delta t(r_2) / (1 - \alpha / r_2) \approx \Delta t(\infty) \approx \Delta t(r_1) / (1 - \alpha / r_1)$$

$$\Delta x(r_2) \cdot (1 - \alpha / r_2) \approx \Delta x(\infty) \approx \Delta x(r_1) \cdot (1 - \alpha / r_1)$$

or

$$\Delta t(r_2) / (1 + \Phi(r_2) / c^2) \approx \Delta t(\infty) \approx \Delta t(r_1) / (1 + \Phi(r_1) / c^2)$$

$$\Delta x(r_2) \cdot (1 + \Phi(r_2) / c^2) \approx \Delta x(\infty) \approx \Delta x(r_1) \cdot (1 + \Phi(r_1) / c^2)$$

Thus, we know exactly how the location-dependent readings for time intervals and lengths must be transformed. Let's treat ourselves to a small calculation: A clock on top of a 22.6 meter high tower emits exactly one beep every second. An identical clock sets at the foot of the tower and measures the time between the beeps. We already know that the lower clock runs slightly slower. What time interval between the beeps does the lower clock measure?

For the ratio  $\Delta t(\text{top}) / \Delta t(\text{bottom})$  we obtain, using the above formulas, the expression

$$\sqrt{(1 - 2 \cdot \alpha_E / (r_E + 22.6)) / (1 - 2 \cdot \alpha_E / r_E)} \quad \text{where } \alpha_E \approx 4.43 \cdot 10^{-3} \text{ m and } r_E \approx 6.373 \cdot 10^6 \text{ m}$$

Entering these values in most calculators will yield simply 1! The two values differ so little that one cannot, even with 10 or 12 digits, see a difference with 1! The differences in small shifts of the gravitational field of the earth are so small that it borders on a miracle that they were already measured in 1960 (experiment of Pound and Rebka, see **14**). In order to calculate the size of the effect, we should therefore not form the ratio  $\Delta t(\text{top}) / \Delta t(\text{bottom})$ , but rather form the ratio of the tiny difference between the two periods to one of the times itself. We would then get something that is smaller than  $10^{-12}$ .

$$\text{Thus we determine } (\Delta t(\text{oben}) - \Delta t(\text{unten})) / \Delta t(\text{unten}) = (\Delta t(r_2) - \Delta t(r_1)) / \Delta t(r_1)$$

It is recommended to use the approximation formula without the radical:

$$\frac{\Delta t(r_2) - \Delta t(r_1)}{\Delta t(r_1)} \approx \frac{\Delta t(\infty) \cdot (1 - \alpha / r_2) - \Delta t(\infty) \cdot (1 - \alpha / r_1)}{\Delta t(\infty) \cdot (1 - \alpha / r_1)} = \frac{\alpha / r_1 - \alpha / r_2}{(1 - \alpha / r_1)} = \alpha \cdot \frac{r_2 - r_1}{r_2 \cdot r_1} \cdot \frac{r_1}{r_1 - \alpha} = \alpha \cdot \frac{r_2 - r_1}{r_2(r_1 - \alpha)}$$

We simply write  $\Delta h$  for the difference  $(r_2 - r_1)$ , and we also bear in mind that we change the result only in the sub part per thousand range if we write  $r_E^2$  for  $r_2 \cdot (r_1 - \alpha)$ . Thus, we get the very simple result

$$\frac{\Delta t(r_2) - \Delta t(r_1)}{\Delta t(r_1)} \approx \alpha \cdot \frac{\Delta h}{r_E^2} \approx 2.47 \cdot 10^{-15}$$

The effect is of the order  $1: 10^{15}$  and it is thus quite understandable that before we did not see a difference in the first 12 decimal places.

We come to a more familiar realm when we do the same calculation with the potential. As before we quickly get (starting with the approximation without the radical) the first term in the following line:

$$\frac{\Phi(r_2) - \Phi(r_1)}{c^2 + \Phi(r_1)} \approx \frac{\Phi(r_2) - \Phi(r_1)}{c^2} \approx \frac{g \cdot \Delta h}{c^2}$$

The first simplification is based on our additional restriction, which is easily met in the gravitational field of the earth:  $\Phi(r)$  is everywhere much smaller than  $c^2$  and can be omitted in the denominator. And for the small height difference of 22.6 m at the earth's surface we can use the good approximation  $g \cdot \Delta h$  for the potential difference in the numerator! With the input  $9.81 \cdot 22.6 / 9 \cdot 10^{16}$  even our calculator is well-behaved giving the result  $2.46 \cdot 10^{-15}$ .

We have just quantitatively mastered a famous test of GTR ! The real trick is, firstly, to formulate the problem in a manner that allows a convenient calculation and, secondly, to cleverly use approximations so the result can actually be determined. More examples will follow.



## G5 Different Speeds of Light ?!

He who can measure lengths and times can also measure speed. Let's consider how to transform the measurements of an observer at rest at a distance  $r$  from the center of our spherical mass into those of an observer at the OFF position.

As in STR we must distinguish whether a speed is observed in the x-direction, i.e., along the field lines (and thus parallel to the velocity of our free falling laboratory), or in the y-direction, i.e., perpendicular to the field lines. The measured time intervals are transformed independent of the direction, while distances are dependent. With the notation  $v_y(r;r)$  we denote the velocity of an object at distance  $r$  from the center of  $M$  in the y-direction, as it is measured from a laboratory at rest at position  $r$ .  $v_y(r;\infty)$  denotes the velocity, that an observer at the OFF position attaches to the same object with the same motion. We calculate:

$$v_y(r;r) = \Delta y(r)/\Delta t(r) = \Delta y(\infty)/(\Delta t(\infty) \cdot \sqrt{1 - 2 \cdot \alpha/r}) = (\Delta y(\infty)/\Delta t(\infty)) / \sqrt{1 - 2 \cdot \alpha/r} = v_y(r;\infty) / \sqrt{1 - 2 \cdot \alpha/r}$$

$$\text{and thus } v_y(r;\infty) = v_y(r;r) \cdot \sqrt{1 - 2 \cdot \alpha/r} = v_y(r;r) \cdot \sqrt{1 - R_S/r} = v_y(r;r) \cdot \sqrt{1 + 2 \cdot \Phi(r) / c^2}$$

It is exactly as in STR: A lateral velocity  $u_y'$  measured locally in the red frame presents itself in the black frame slowed by our radical term:  $u_y = u_y' \cdot \sqrt{\quad}$  (see D5)

We use the approximations of G3 for  $\sqrt{1 - 2 \cdot \alpha / r}$  to obtain the simpler expression

$$v_y(r;\infty) \approx v_y(r;r) \cdot (1 - \alpha/r) = v_y(r;r) \cdot (1 + \Phi(r) / c^2)$$

Applying the same for velocities in the x-direction produces a different result:

$$v_x(r;r) = \Delta x(r)/\Delta t(r) = (\Delta x(\infty)/\sqrt{1-R_S/r}) / (\Delta t(\infty) \cdot \sqrt{1-R_S/r}) = (\Delta x(\infty)/\Delta t(\infty)) / (1-R_S/r) = v_x(r;\infty) / (1-R_S/r)$$

$$\text{and thus } v_x(r;\infty) = v_x(r;r) \cdot (1 - R_S/r) = v_x(r;r) \cdot (1 - 2 \cdot \alpha/r) = v_x(r;r) \cdot (1 + 2 \cdot \Phi(r)/c^2)$$

Here our radical term is squared and thus the root disappears! In this case we do not need the approximations.

Elementary, but somewhat cumbersome calculations [27-135f] result in an approximation which is good for all directions. Let  $\vartheta$  denote the angle between  $v$  and the field lines:

$$v(r;\infty;\vartheta) \approx v(r;r;\vartheta) \cdot (1 - \alpha \cdot (1 + \cos^2(\vartheta)) / r)$$

The term  $1 + \cos^2(\vartheta)$  is equal to one, when  $\vartheta = 90^\circ$ , that is, when the motion is in the y-direction, and it is equal to 2 when  $\vartheta = 0^\circ$  and  $\vartheta = 180^\circ$ , that is, when it is in the x-direction. We could have actually guessed this formula ...

Thus, as seen from a distance, speeds slow in the vicinity of large masses. That is not surprising, since time itself seems to slow down. But now the hammer: This holds for the speed of light, too !!

If we measure the speed of light in a vacuum in any direction in a laboratory resting at position  $r$ , we obtain the default value of  $c(r;r;\vartheta) = c_0 \approx 3 \cdot 10^8$  m/s. From the viewpoint of an observer at position OFF, however, we obtained this value with our possibly-shortened yardsticks and our certainly-slowed clocks. Thus for an observer from OFF, the light at the location  $r$  in the gravitational field of mass  $M$  must spread out with the slower and direction-dependent speed  $c$ , as follows:

$$c(r;\infty;\vartheta) \approx c(r;r;\vartheta) \cdot (1 - \alpha \cdot (1 + \cos^2(\vartheta)) / r) = c_0 \cdot (1 - \alpha \cdot (1 + \cos^2(\vartheta)) / r)$$

This means that for an observer at OFF  $c_0$  represents only the upper limit of the speed of light for gravity-free space. In the vicinity of masses, light (as observed from OFF) is slower. Meanwhile, local observers always measure at their position the known value of  $c_0$ . Locally the world is everywhere Lorentzian ...

We make a note of the speed which light has in the vicinity of large masses for an observer in the OFF position:

$$\begin{aligned}
 c_x(r; \infty) &\approx c_0 \cdot \left(1 - \frac{2 \cdot \alpha}{r}\right) \\
 c_y(r; \infty) &\approx c_0 \cdot \left(1 - \frac{\alpha}{r}\right) \\
 c(r; \infty; \delta) &\approx c_0 \cdot \left(1 - \frac{\alpha}{r} \cdot (1 + \cos^2(\delta))\right)
 \end{aligned}$$

Gravity acts (observed from a distance) on light like an index of refraction! The stronger the field is the slower light advances. The index of refraction depends not only on the position  $r$ , but also on the direction  $\delta$ . If the index of refraction changes then light no longer spreads in a straight line – except when the light beam runs exactly along a field line from or towards the center of  $M$ . In all other cases, the phenomenon of refraction occurs. We are now prepared for the calculation of the experimental result which made Einstein a celebrity of the first rank over night: the deflection of light at the solar limb, which can be photographed only during a solar eclipse (12).

All velocities indeed transform themselves according to our formulas above, we need only replace  $c_0$  by the locally measured velocity  $v(r;r)$ . This means that the quarter of a satellite, which is closest to the earth moves slightly slower than the part that lies further away! If, in a row of four people, the ones on the right are always a bit faster than those on the left, then the direction in which they march will gradually turn to the left. Instead of the free falling (or flying) satellite moving 'straight', it will therefore run in an orbit around the earth! In just this way Einstein explains the effect of gravity – without using the concept of force. The 'distortion' of the metric of space and of space-time itself produce 'inertial trajectories' just as we know them from Newtonian mechanics.

If we describe everything from the perspective of the observer in OFF, then we can do away with 'shortened yardsticks' and 'slow clocks' and infer all phenomena just from this refraction in the vicinity of masses. In fact, however, we are sitting in the midst of such gravitational fields, and our atomic clocks today show directly the effects demanded by Einstein (15, 16 and 17). It would be in any case somewhat elitist or aloof, just to deal with the world only from the OFF position ...

Epstein devotes four pages to the theme of "gravitation by refraction" [15-160ff] - recommended to the interested reader. A very nice relevant illustration can be found at the end of the next section.

We can now perhaps even understand the pithy comment of Misner, Thorne and Wheeler which summarizes general relativity theory in an unsurpassed manner:

"Matter tells space how to curve, and space tells matter how to move." 'phone book' [27-0005]

With 'space' designating 4D space-time, of course ...

## G6 Problems and Suggestions

1. Slowly pour colorless lemon syrup into a glass, which is already half filled with water. Aim the beam of a laser pointer through the glass and watch as the locally different indices of refraction provoke varying local light speeds and thus direction changes!
2. Correct and clarify the statement in [11] on page 124! It deals with the contraction of yardsticks in the vicinity of large masses.
3. Visit the website [www.zarm.uni-bremen.de/index.htm](http://www.zarm.uni-bremen.de/index.htm) to the drop tower in Bremen.
4. Calculate the Schwarzschild radius of the moon, the earth and the sun. Compare them with the effective radius of the spherical body, ie determine the ratio  $R_s / R$ .
5. Calculate the ratio  $R_s / R$  for an atom and a nucleus. Must the effects of GTR be taken into account in nuclear physics?
6. Read the Einstein biography [31] from Thomas Bührke !
7. Look for websites or documents describing the experiments of Eötvös and Dicke on the equality of inertial and gravitational mass, and study the basic idea of these experiments.
8. The American physicist Richard P. Feynman proposed the following to illustrate length contraction: Consider a huge stove, which is heated so that it is cool in the middle and gets warmer the further you go from the center. If done correctly then yardsticks in the radial direction will have the correct length due to thermal expansion! What is the power of this idea - and what are its flaws ?
9. Devise appropriate clocks for Feynman's hotplate version of the Schwarzschild metric.
10. How many seconds does a lifetime of 80 years have, if one lives in the Maldives or if one lives in the High Andes at an altitude of 4000 m ?
11. A whole new kind of matter has been found in a meteorite in which inertial and gravitational mass differ. How can we ever ascertain that? What would be the consequences for the gravitational theory of a) Newton b) Einstein
12. What would be 'rectilinear' in a gravitational field in which light rays are bent?
13. Show that a clock on board a satellite orbiting in a circle with radius  $r$  in a Schwarzschild field of mass  $M$  runs slower by the factor  $(1 - 3 \cdot \alpha / (2 \cdot r))$  than an identical clock at OFF position. You need both STR and GTR !
14. Calculate the quotient of the speed(s) of time of two clocks, which are near earth's surface at locations with a height difference of  $\Delta h$ , by replacing gravity with a rocket of length  $\Delta h$  which is being accelerated with value  $g$  in a gravitation-free field. Set the initial speed to 0 and take into account the Doppler effect! When the signal arrives at the top, the clock at the top is already moving a bit faster than the clock at the bottom was when the signal was emitted ...



Some of the most beautiful 'Einstein rings' from the collection of the Hubble Space Telescope. They arise because the gravitational field of the central bright orange galaxy acts like a lens on the beam of the blue light of a far distant source (usually a quasar). We then see the same object all around the edge of the foreground galaxy. Also the intensity of the light of the background object is massively increased.

Einstein theoretically predicted such gravitational lenses in 1936, but he was rather pessimistic about the possibility of this effect in fact ever being observed. It did take 60 years ...

--> <http://hubblesite.org/newscenter/archive>, key words 'gravitational lens'



Spiral Galaxy NGC 7424  
(VLT MELIPAL + VIMOS)

ESO PR Photo 33c/04 (1December 2004)

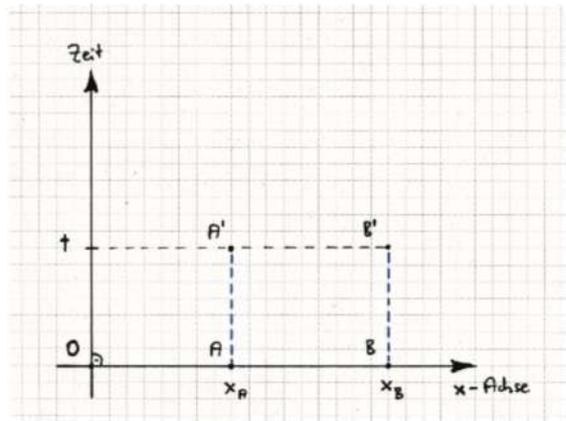
© European Southern Observatory

## H Epstein Diagrams for the General Theory of Relativity

We learn from Epstein diagrams how objects spontaneously begin to fall, and how clocks tick more slowly under the influence of stronger gravitation. To do so we need only to bend our familiar Epstein STR diagrams a bit ... The nice thing is that we can maintain the basic axiom: Everything always moves through space-time at the speed of light. Out of this arises a new variant of the twin paradox. As usual we'll investigate this quantitatively. We then interpret the results using the principle of maximum proper time. The fifth section presents Epstein diagrams in an elegant rolled version. Finally, we let Epstein show us how the curvature of space alone bends the inertial trajectories of fast objects. Here we anticipate qualitatively some experiments that we will handle quantitatively in I.

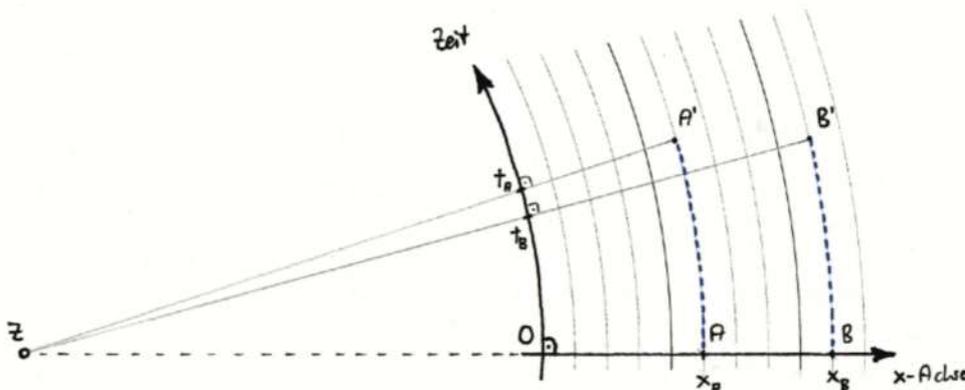
# H1 Gravitation and the Curvature of Space-Time

Below is a space-time diagram, as we learned to draw with STR in section C:



Clocks are placed at the positions  $x_A$  and  $x_B$  and both have been set to zero in A and B. Through the projection of the space-time position of the clocks on the time-axis, we can determine what time the clocks show at any position. One can use the time-axis to read the elapsed proper time for each object since the start of an operation.

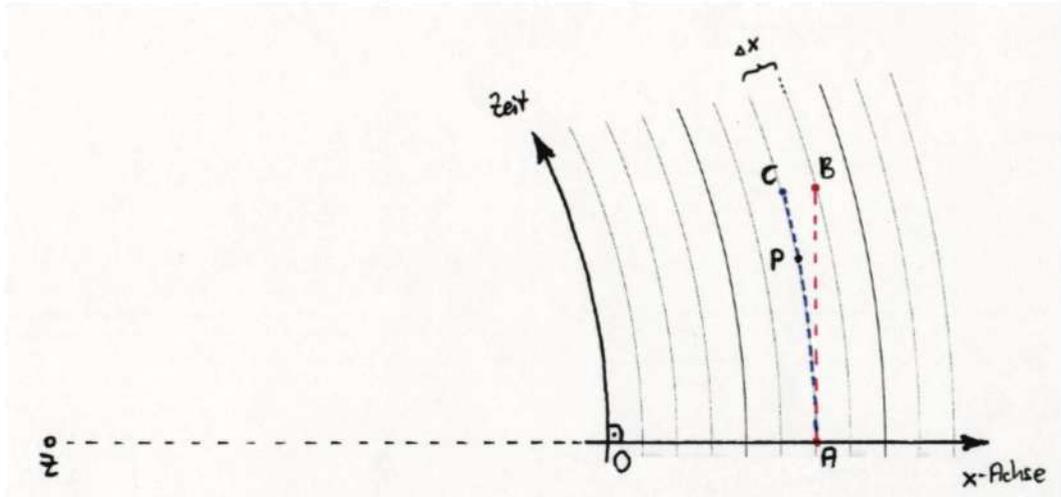
Let the gravity at position  $x_B$  be stronger than at  $x_A$ . What can be done so that along the path  $BB'$  time runs a bit less quickly than along the path  $AA'$ , which is of *exactly the same length* in space-time? Epstein shows us the simple yet elegant solution: we only need to bend the time-axis a bit away from B to produce the desired effect!



Objects that linger at point  $x_A$  move on a circle centered at Z with radius  $ZO + x_A$ . The corresponding applies to stationary objects at  $x_B$ . It is important that the two arc segments  $AA'$  and  $BB'$  are exactly the same length. This arc length specifies how long the process lasted for an observer in the OFF. We can read the elapsed proper time for A or B, by projecting  $A'$  or  $B'$  vertically on the now circular time-axis. We need only link  $A'$  and  $B'$  with Z. The clock at  $B'$  apparently shows less time than the one at  $A'$ !

The curvature of the time-axis, i.e., the inverse of the radius  $ZA = ZA'$ , is critical for the strength of the effect at point  $x_A$ . This curvature is a function of the distance  $r$  from the center of the central mass. This is to the right of  $x_B$ , because gravity is stronger at  $x_B$  than  $x_A$ . If the curvature is zero, i.e., the radius  $ZO$  is infinitely large as in the first Epstein diagram above, then there is no effect from gravity and we are back in the STR.

We have now described the behavior of static objects in a gravitational field. As examples of such objects, imagine an apple in the Netherlands or a pine cone in the Swiss Alps each of which is hanging fixed on its branch. But what happens when the connection between the stem and the fruit breaks? The Epstein diagram holds the answer to this question:



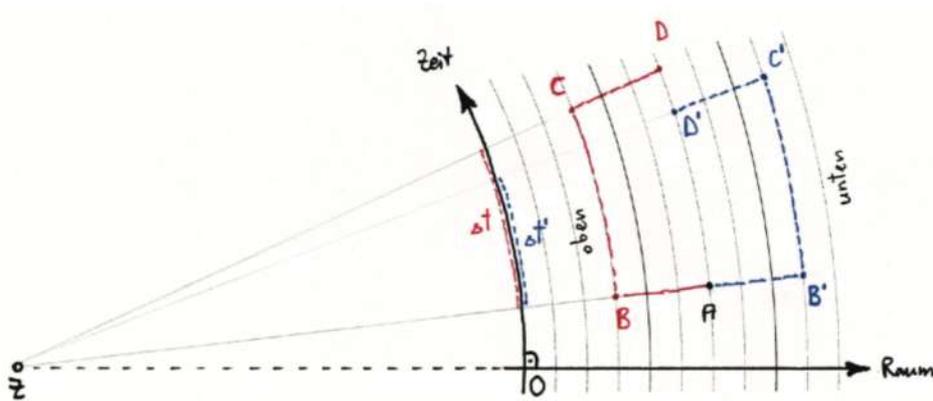
Apple and branch are located at time zero at point A in space-time. Just then the connection between the two is cut. The branch moves up as fixed from A to C, while the apple moves along the tangent to arc AC, i.e., along the red dotted line and arrives at B! For the freely falling apple, there is no longer gravity. For the apple it is rather as if the tree is being prevented by the ground from following an inertial trajectory. After a short time, a distance  $\Delta x$  opens between the apple and the branch, whereby the apple has moved in the direction of the center of mass. The arc length AC and the segment length AB are again exactly the same length.

The branch traverses the arc AC proportional to time. At point P on the arc, the angle between the tangent to the circle at P and the red line AB is equal to the angle PZO. This angle indicates, however, how quickly the apple is moving away from the branch at a given moment, that is, the relative velocity of the apple and branch. Thus we can even read Galileo's law for falling bodies from the diagram: Regardless of its mass the apple has a speed proportional to the elapsed time! Recall, however, that this law has a limited scope:  $v$  must be much smaller than  $c$ !

The angle between two curves in the space-time diagram still indicates a relative velocity. More precisely:  $\sin(\phi) = v/c$  ! The maximum angle is still  $90^\circ$ . Light moves, as usual, perpendicular to the time-axis. However, in the case of apples falling at the earth's surface, a quantitative analysis of the intermediate angle is tricky, as section **H3** will show. But before doing so we would like to use our bent space-time diagrams to present a new variant of the twin paradox.

## H2 Twins Again, With Different Rates of Aging

A picture is worth a thousand words:



Epstein's twins, Peter and Danny, decide they no longer want to be the same age. Since time 'deep' in a gravitational field runs more slowly than 'higher up' in the field they decide while having tea that Peter will spend some time on the second floor while Danny hangs out in the cellar. At point A, they begin to implement this plan. The lines AB and AB' and CD and C'D' are of equal length. But also the arcs BC and B'C' are of equal length, so the two will meet again at the same time on the first floor, i.e., at the place of A, D' and D. Peter is now really a bit older than Danny!

What does it mean, however, 'to meet again at the same time'? We assume again the position of an observer in OFF. Thus, for us the whole experiment elapses in time  $AB + BC + CD = AB' + B'C' + C'D'$ . For the red (Peter) the time elapsed is given by the projection of the arc BC on the time axis (i.e., the red  $\Delta t$ ), while for the blue (Danny) even less time has elapsed. For him it is given by the projection of B'C' on the time axis (i.e., the blue  $\Delta t'$ ).

Did you notice that Peter and Danny were very quick at climbing stairs? How fast were they moving? Well, the whole story would also work if they took it somewhat more relaxed. The drawing would, however, be a bit more complicated.

The Swiss lawyer, philosopher and chansonnier Mani Matter has written a song about a boy running 'as fast as lightning':

there is a boy let's call him Fritz, there is a boy let's call him Fritz  
that can run as fast as a blitz, that can run as fast as a blitz.

he runs that outrageous athlete, he runs that outrageous athlete  
so fast you can not see his feet, so fast you can not see his feet.

and 'cause he never ends his race, and 'cause he never ends his race  
no one has ever seen his face, no one has ever seen his face.

so even I, the bard, agree, so even I, the bard, agree  
perhaps the lad can't even be, perhaps the lad can't even be.

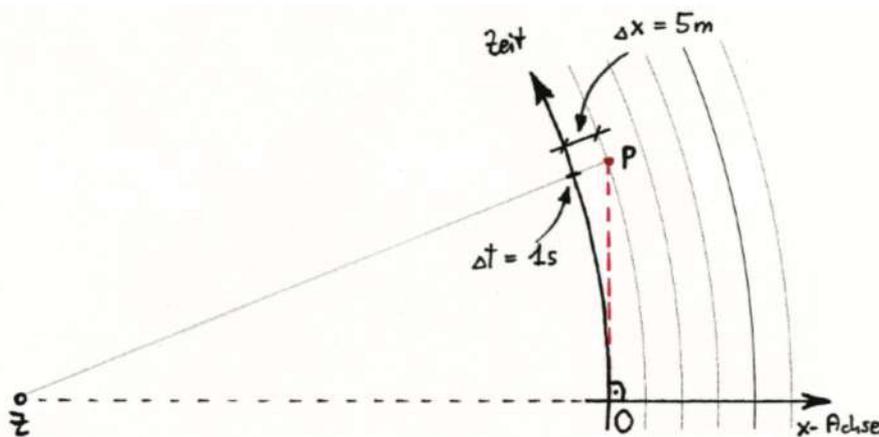
Lyrics and tune by Mani Matter [46-41], translated from the original Schwiizertütsch (Swiss German) by David Eckstein and Samuel Edelstein

Fritz becomes a victim of Ockham's razor! (see problem 1)

### H3 A Quantitative Consideration

How far does an apple fall in one second from initial rest? Assuming for  $g$  the rounded value of 10, we get from  $\Delta h = 0.5 \cdot g \cdot \Delta t^2$  a distance of 5 m. But one second of time on our scale diagram corresponds to a length of 300,000,000 m! If a second on the diagram has a length of 3 cm, a distance of 5 m in the diagram has a length 0.5 nm (nanometers) - which corresponds to just a few atomic diameters!

Let's calculate the length of the radius of the time axis (or, the inverse value, its curvature) when the space-time diagram is adjusted to the strength of the gravitational field at the Earth's surface:



We express all lengths in our space-time diagram in meters. The leg  $OP$  of the right triangle  $ZOP$  has, to a good approximation, the length  $c \cdot 1s = 3 \cdot 10^8$  m (the final result will show that this approximation is actually extremely precise ...). The Pythagorean Theorem now gives us:

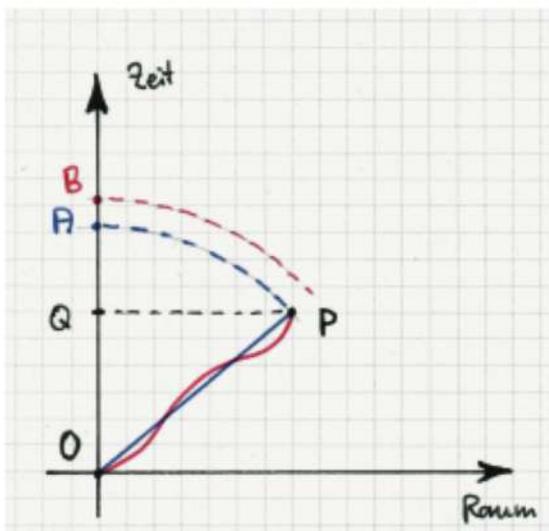
$$ZP^2 = ZO^2 + OP^2 \quad ; \quad (ZO + 5)^2 \approx ZO^2 + (3 \cdot 10^8)^2 \quad ; \quad ZO^2 + 2 \cdot ZO \cdot 5 + 5^2 \approx ZO^2 + 9 \cdot 10^{16}$$

$$10 \cdot ZO \approx 9 \cdot 10^{16} - 25 \quad ; \quad ZO \approx 9 \cdot 10^{15} - 2.5 \quad \text{and thus} \quad ZO \approx 9 \cdot 10^{15}$$

If the curvature of the time axis fits the strength of gravity on earth's surface, then the radius  $ZO$  must be  $9 \cdot 10^{15}$  meters, which is almost exactly one light year! For comparison: the nearest fixed star is about 4 light years away from us and the semi-major axis of Pluto's orbit is only  $5.9 \cdot 10^{12}$  m. The correct radius is therefore immense, and the correct curvature is minuscule. We are talking about a really weak gravitational field – whose curvature cannot be detected by the naked eye. But it is this tiny space-time curvature on earth's surface that causes freely moving objects to fall and that makes climbing stairs so strenuous!

Adam Trepczynski has created a Shockwave animation, which allows one to play with Epstein space-time diagrams - with and without gravity:  
[http://www.relativity.li/uploads/flash/epstein\\_space\\_time.swf](http://www.relativity.li/uploads/flash/epstein_space_time.swf)

## H4 Principle of Maximum Proper Time



No matter which path a clock travels from  $O$  to  $P$  through space-time - the time elapsed on the clock will always be  $OQ$  as measured by any inertial observer. The red and blue path, however, differ in that our 'black' inertial observer measures different elapsed times on his own clocks for the blue and the red path. For the blue path, it is simply the length of the segment  $OP = OA$ . But since everything always travels the same distance through space-time, we need only stretch the red path to find the corresponding elapsed time  $OB$  for black.

The proper time  $OQ$  that elapses on the clock is thus independent of the chosen path, but not, however, the quotient

formed from this proper time and the time elapsed for an observer from any other inertial frame. For the straight blue path from  $O$  to  $P$  the quotient  $OQ / OP = OQ / OA$  has its maximum value, compared to all other paths from  $O$  to  $P$  as seen from the arbitrary chosen black system. The quotient is maximal because the denominator  $OA$  is minimal!

The name "principle of maximum proper time" is a bit misleading in this context since what is actually maximized for the blue path is the ratio  $\Delta\tau / \Delta t$ , when we, like Epstein, denote proper time with  $\tau$  and coordinate time with  $t$ .  $\Delta\tau$  is measured on a single clock moving from  $O$  to  $P$  and influenced by gravitation and velocity, while  $\Delta t$  is measured on a clock of the distant observer in OFF, who in GTR really has a special position. While in STR we have given preference to symmetric presentations, we are now forced by gravitation to use the point of view of the distant observer in the OFF to compare other measurements.

**Principle of maximum proper time: If no force is acting on a body, it moves from  $X$  to  $Y$  through space-time along a path for which the ratio  $\Delta\tau / \Delta t$  is maximized.**

In the STR, these paths are straight lines. Any other path through space-time is longer and thus increases the denominator  $\Delta t$ . Before we look at an example of the behavior in GTR, I would like to emphasize that here we do *not* consider the path that light takes from one location  $A$  to another location  $B$  through space - we will later. Here we consider "free fall lines" through space-time.

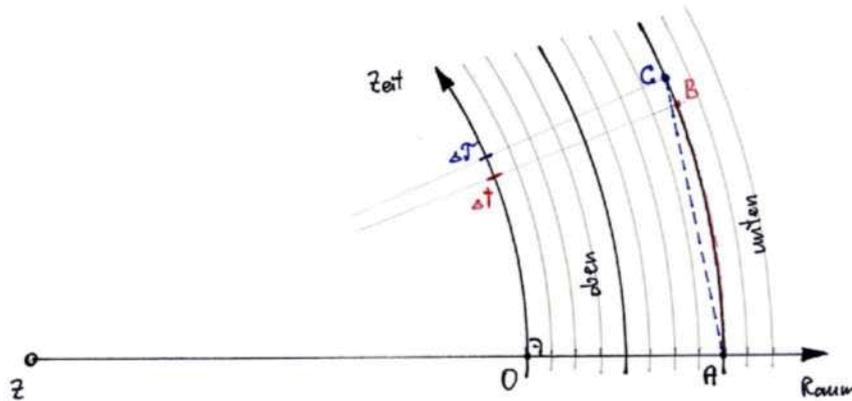
Now for an example:

What is the force-free path from a point  $A$  on the surface of the earth which leads 2 seconds later to the same place? It is a vertical throw with an initial speed of 10 m/s. During the first second the object rises, ever more slowly, to a height of 5 m and in the second second it falls freely from its height back to the starting point. We know the equations of motion and the path in space from Newtonian physics:

$$h(t) = v_0 \cdot t - 0.5 \cdot g \cdot t^2 \approx 10 \cdot t - 5 \cdot t^2 \quad \text{und} \quad v(t) = v_0 - g \cdot t \approx 10 - 10 \cdot t$$

Why does *this* movement satisfy the principle of maximum proper time, why is it not more advantageous simply to remain on the ground or to rise to a height of 20 meters?

First consider the related Epstein diagram:



According to Epstein's dogma, let the straight blue path from A to C have the same length as the red arc from A to B on the circle. The proper time interval  $\Delta\tau$  belonging to the force free flight from A to C is obviously longer than the time interval  $\Delta t$  measured with a clock at rest, moving through spacetime from A to B on an isotropic arc.

Why is the blue path more advantageous? Because higher up the clock is running faster! Why does it not then go even further? Because then it must go up and down so fast that the time dilation due to the high velocity would negate the advantage of the greater height! GTR advocates height, STR advocates small speeds - and the optimal compromise is our vertical throw!

$$\frac{\Delta t(h) - \Delta t_0}{\Delta t_0} = \frac{g \cdot \Delta h}{c^2}$$

What time increase does the vertical throw bring? We already deduced in **G4** how the time difference depends on the height difference:

$$\int_0^2 \frac{g \cdot \Delta h}{c^2} \cdot dt = \frac{g}{c^2} \cdot \int_0^2 (10 \cdot t - 5 \cdot t^2) \cdot dt = \dots = \frac{g}{c^2} \cdot \frac{20}{3} = \frac{200}{3 \cdot c^2}$$

We must sum the increase per unit of time for the entire flight time of 2 seconds. The corresponding integral is harmless

This is the *additional* elapsed proper time in seconds due to the "hop". We must balance this with the loss of proper time due to the velocity. According to the STR

$$\frac{\Delta t_0 \cdot \sqrt{1 - \frac{v^2}{c^2}} - \Delta t_0}{\Delta t_0} = \sqrt{1 - \frac{v^2}{c^2}} - 1 = \left( 1 - \frac{v^2}{2 \cdot c^2} - \frac{v^4}{8 \cdot c^4} - \dots \right) - 1 = -\frac{v^2}{2 \cdot c^2}$$

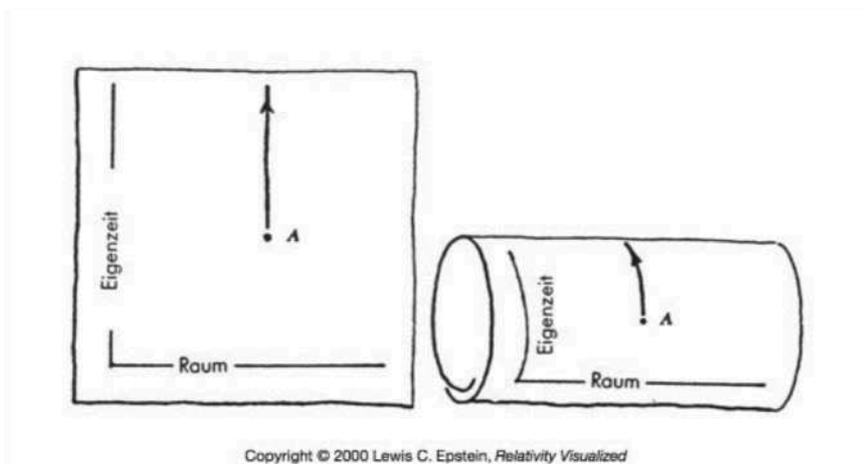
We know that  $v(t) \approx 10 - 10 \cdot t$  and can therefore again integrate from 0 to 2 over time  $t$ . This integral is also quite harmless:

$$\int_0^2 -\frac{v^2}{2 \cdot c^2} \cdot dt = \frac{-1}{2 \cdot c^2} \cdot \int_0^2 (10 - 10 \cdot t)^2 \cdot dt = \frac{-100}{2 \cdot c^2} \cdot \int_0^2 (1 - t)^2 \cdot dt = \dots = \frac{-100}{2 \cdot c^2} \cdot \frac{2}{3} = \frac{-100}{3 \cdot c^2}$$

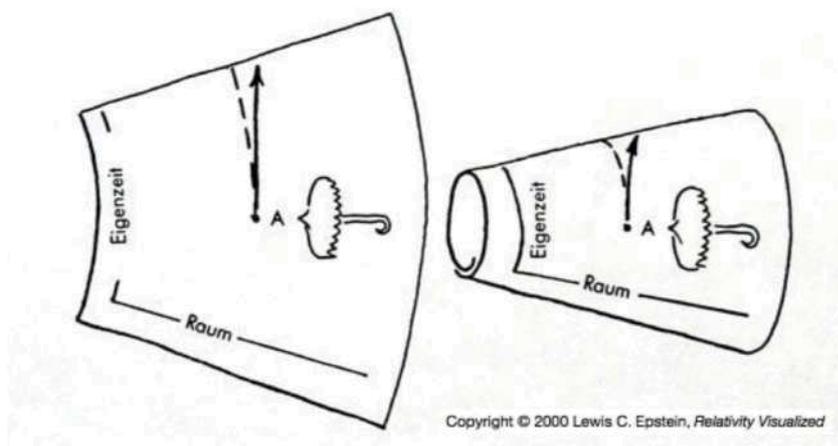
The loss from STR is thus only half as large as the gain from GTR and the overall positive balance is  $+100 / (3 \cdot c^2)$  seconds. In problem 4, you will be asked to show that the increase of proper time is greatest for *this* parabola; that for higher paths the loss due to the STR increases faster than the gain due to GTR - and vice versa. The solution of Galileo and Newton corresponds exactly to the parabola which fulfills the principle of maximal proper time!

## H5 Epstein Diagrams - Flat or Rolled

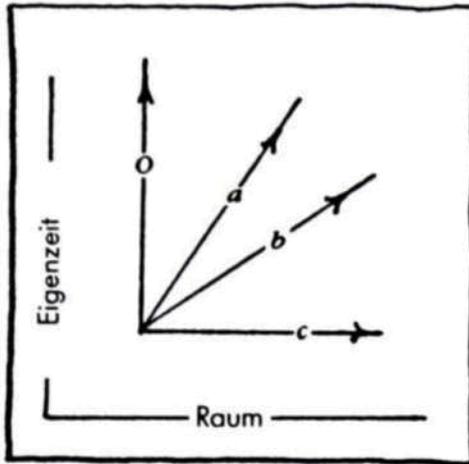
For periodic processes such as the up and down oscillations of a spring pendulum in a gravitational field or the somewhat fictitious free fall through a tunnel which traverses the center of the earth, the moving object repeatedly returns to the same place. Such processes can be very beautifully depicted in a rolled version of an Epstein diagram. The first picture shows a stationary object in both a flat and in a rolled Epstein diagram. The time axis is not curved; there is no gravity at work; we are in the realm of STR:



What do we get when we roll the curved diagram of section H1? A lampshade, or more scholarly, a truncated cone:



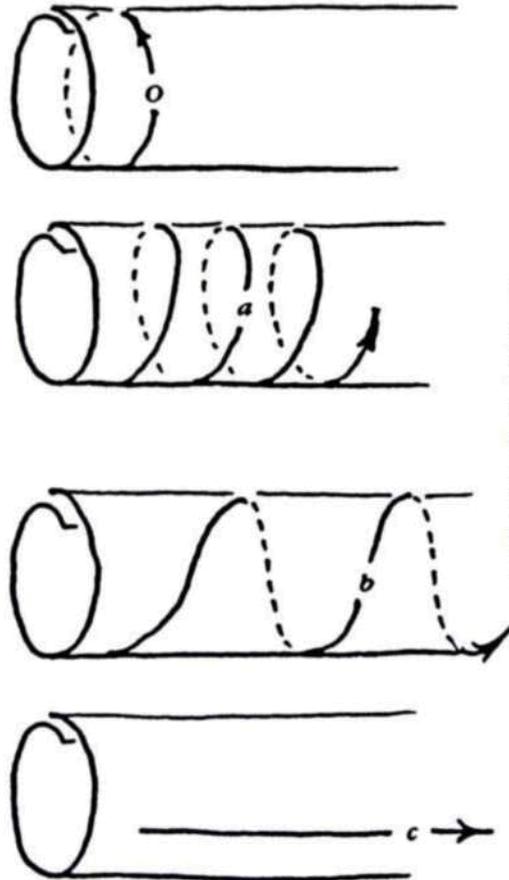
In his diagrams [15-146ff] Epstein occasionally suggests by using glasses or parasols the 'down' direction of the gravitational field (all diagrams in this section are taken from Chapter 10 of his book). We once again nicely see how an object that is not held in place by force moves in a straight line through space-time toward the 'bottom'. The angle  $\phi$  between this 'fall-line' and the circles on the surface of the cone, which are at a fixed location and centered on the axis of symmetry of the cone, continually increases.  $\sin(\phi) = v/c$  is still correct. It just indicates that the relative speed between fixed points and the free-falling object is increasing - just as we have found in section H1.



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Let's consider the paths of various fast moving objects in gravity-free space-time. We are familiar with the representation in a plane: *o* lies at the origin, *a* is pretty fast, *b* is even faster, and *c* gives the behaviour of a photon.

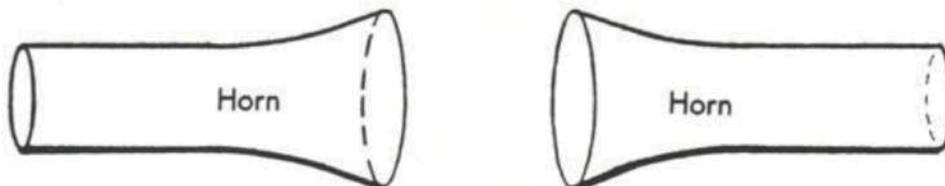
To the right, all four speeds are represented in the rolled version. We get four simple paths on a cylindrical surface.



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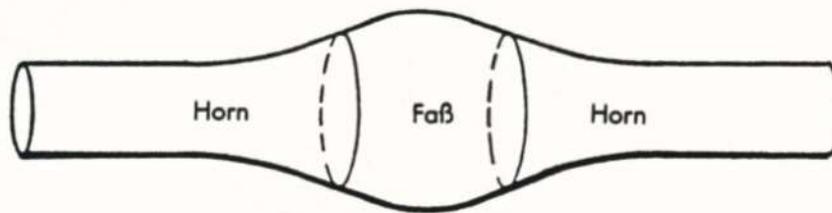
We now could study how this is presented in a homogeneous gravitational field, with  $g$  and thus the curvature of space-time being constant. A lampshade is a good representation of the *local* situation at a given distance from the center of the field generating mass. But large spatial movements can, however, not be represented!

Which is the corresponding solid of revolution, if the curvature is increasing with proximity to the Earth's surface - or, in other words, when an stationary orbit continually claims more proper time? The answer: Something that looks like the bell of a trombone or an ear trumpet. Epstein calls this body a horn.



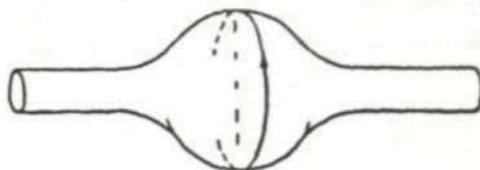
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This diagram presents a large section of the  $x$ -axis on both sides of the earth together with the "rolled" time axis. The situation in the earth's interior must be presented in the gap in-between. There, however, gravity (and hence the curvature) decreases linearly toward the earth's center. Therefore, between the two horns a sphere with detached polar caps must be added.

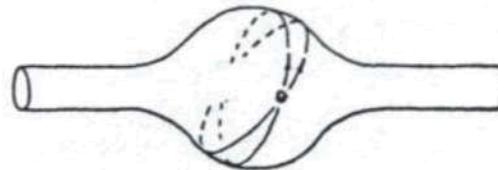


The 'barrel' in the middle is actually an excised sphere, as we will presently prove. Imagine a cylindrical tunnel directly through the earth's center from Switzerland to, let's say, New Zealand. The tunnel axis is identical to our x-axis and the origin corresponds to the center of the earth. We can now, starting from any point on the x-axis, send an object, with or without an initial velocity, on a journey through the earth along the x-axis.

Objects falling near the earth swing like a spring pendulum back and forth around the center of the earth. In the interior of the earth the gravitational force obeys Hook's Law:  $F = -k \cdot \Delta x$ . The oscillation period - as with the spring pendulum - is independent of the initial degree of deflection from the resting position, and thus the paths rolled into our space-time diagram must be the same length, regardless of the initial velocity! This is exactly satisfied if the 'barrel' between the two horns is a section of a sphere! The 'straight' paths, which correspond to free-falling are only on a sphere's surface always closed, and also have in all cases the same length per orbit:

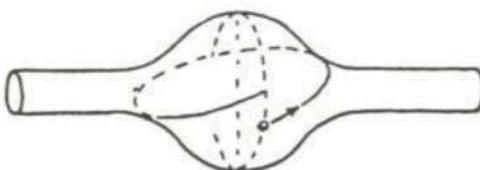


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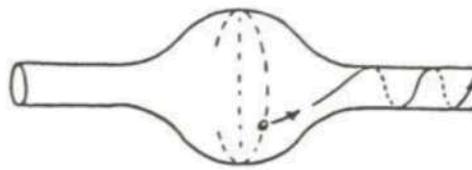


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But throw an object with a positive initial speed in this tunnel, it will go beyond the spherical region and will also take somewhat more proper time per orbit than the pendulum inside the earth (sketch below on the left). If the initial velocity is even larger than the escape velocity of about 11.2 km/s, the object will escape (sketch below on the right):



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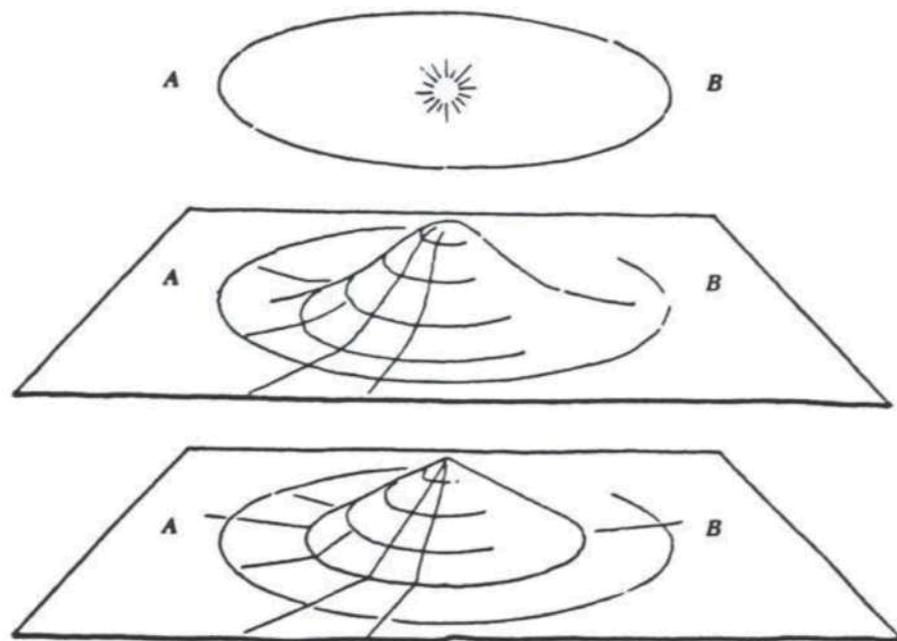
So much for the curvature of space-time as the cause of effects which we normally blame on forces in Newtonian physics. But that is only half the story: Even the metric of space itself is distorted. We will study these effects - again with Epstein - in the next section.

Adam Trepczynski has also produced a Shockwave animation of the rolled space-time diagrams of Epstein which he has kindly made available: <http://www.relativity.li/uploads/flash/gravitation.swf>

## H6 Gravitation and the Curvature of Space

We have already seen in **G4** that the metric of space in the vicinity of a gravitational mass no longer obeys the laws of Euclid. For example, local yardsticks measure the diameter of the earth as slightly larger than its circumference divided by  $\pi$ . The effect in a weak gravitational field such as that of the earth is again very small, but in the vicinity of the sun it is nowadays easily experimentally demonstrated.

Consider again the diagram in **G4**. The dimple, or as Epstein says, the bump can help uncover the behavior of these metrics in an additional dimension that is not related to the z-direction. Epstein gives [15-165ff] detailed instructions on how you yourself can build such a model of a plane through the center of the sun (i.e., the ecliptic). The continuous changes in curvature are ignored for simplicity's sake and the whole bump is represented by a cone:

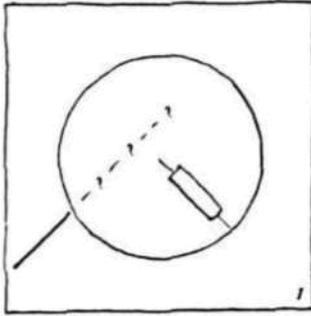


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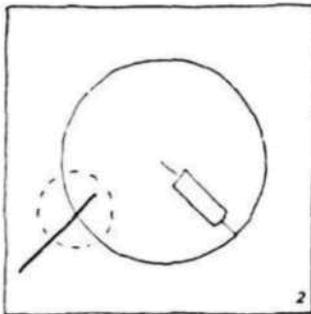
This simple model shows qualitatively all of the effects which according to the GTR have their source in the non-Euclidean metric of space!

I trust the reader can craft such a cone themselves with no additional instructions. Keep in mind that it must be possible to 'open' the cone and spread it flat on a table and then afterwards again reform it into a cone. Therefore it is preferable to use the milky variety of scotch tape rather than the clear ...

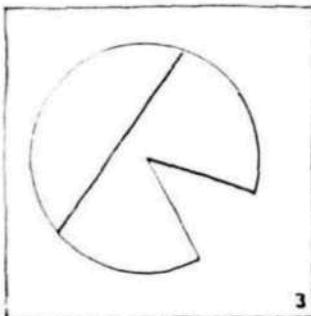
With this model we are investigating – following always [15-165ff] Epstein - what effect such a space bump has on the path of a light beam (further applications will follow in the next section I). It is truly awesome how Epstein using such simple means, accurately and clearly shows the effects of space curvature!



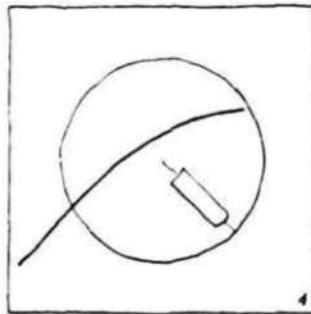
A beam of light approaches our space bump. Which path will it follow, when it enters the field of the non-Euclidean metric?



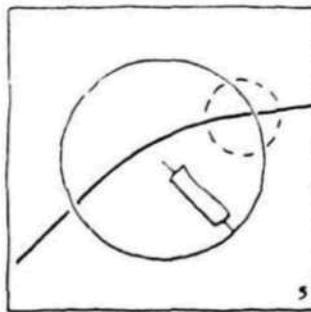
The transition from the diagram's flat surface to the curved cone is elegant: Remember that the bump rises softly from the flat surface and that the edge does not really exist! Flatten the cone locally just a bit (dotted circle) and extend the linear beam inside the dashed circle.



The beam will continue to spread out in a 'straight' line. But what does this mean on the surface of a cone? You can easily answer this if you uncoil the cone onto the diagram's flat surface. Extend the small, straight piece you already have inside the dashed circle of the cone mantle until the beam of light again leaves the cone.



Return the cone to its 3D shape. Put it in exactly the same position as it already had in Figure 2. Thus we have – as viewed from above - the conditions of a distorted geometry of space produced near a large mass. We will now see which path the light beam follows around the center of this mass.



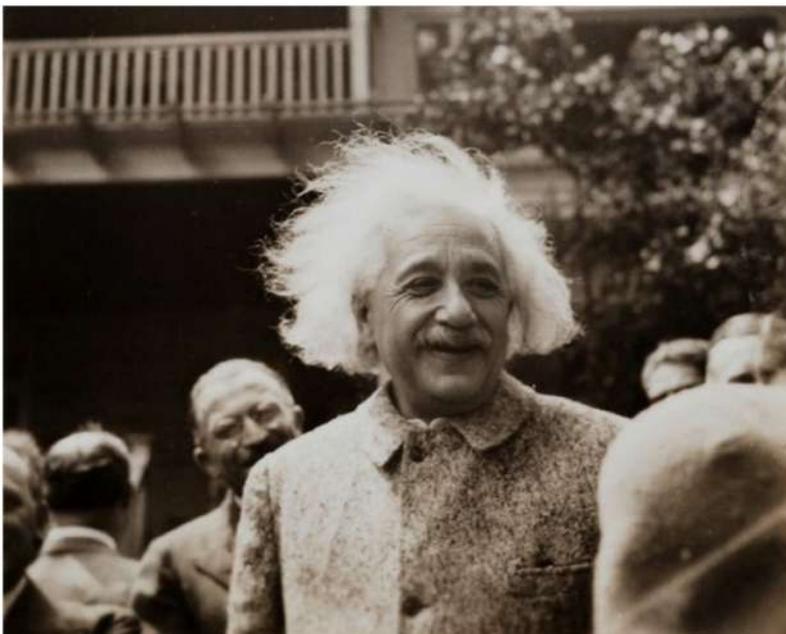
Exactly as in step 2 we now construct a further path. Press the cone in the dotted circle flat, and extend the direction of the path at the cones edge into the plane - done!

Light particles in a gravitational field fall (like everything else) in the direction of the mass center due to the curvature of space-time. Since this is independent of the mass of the falling particle, one was able to calculate this effect even before one knew the inertial mass of a light particle. The 27-year-old Johan Soldner submitted in 1803 an essay based on Newton's theory in which he calculated the deflection of a light beam passing the edge of the sun. The result of his calculation was an angle of 0.875 arc sec. This is precisely the value Einstein obtained in 1911 when he tested his newly conceived theory. In 1916 the 'completed' GTR provided a prediction of 1.75 arc sec, i.e., twice the value. This angle was now large enough that the possibility existed to clearly measure it from a photograph of stars in the sun's vicinity during a solar eclipse. This was achieved in 1919 by two excursions of the 'Royal Astronomers' under the direction of Arthur Stanley Eddington.

It was space curvature that was still lacking in the theory of 1911. Both curvatures (space-time and space-space) contribute almost the same and double the total effect. So a difference to the predictions of Newton's theory arose and the result of the measurement was in favor of the GTR and against Newton. Two theories may be structured differently – but if they make the same predictions for all phenomena, it cannot be decided through an experiment which is preferable. Fundamentally an experiment can only *disprove* a theory - never prove it.

There is however a remarkable difference between the curvature of space-time and that of space: The spatial curvature only affects objects that are moving through space, it has no influence on objects at rest! In particular, the curvature of space cannot cause an object to begin falling. But if it falls then it affects its trajectory. That the apple begins to fall is attributable to the curvature of space-time alone.

Epstein offers an interesting analogy [15-174, footnote]: It is like the effect of electric and magnetic fields on charged particles. The electrostatic force is there and works, regardless of the speed of the particle. The magnetic field has no effect on a charge at rest; the Lorentz force is proportional to velocity. Here, unfortunately, the analogy stops: The effect of spatial curvature on the path of falling objects does *not* depend on their velocity, it is important only that they have a path through the area to follow. However, velocity has a big impact on how long the curvature of space-time has an effect. Here the analogy fits again, since this holds for a charged particle in an electric field as well.



Einstein, about 1945

## H7 Problems and Suggestions

1. Read the Wikipedia entry on what is meant by "Occam's razor" or "Ockham's razor". Then think about the ether, parallel universes and the extra 6-8 dimensions that modern string theories postulate.
2. Calculate as in **H3** the radius of curvature of a space-time diagram which depicts the situation on the surface of the sun.
3. Draw flat, unrolled Epstein diagrams for the 4 paths (o, a, b, c) of section **H5** as seen by an observer at rest at the origin.
4. Calculate the net gain of proper time according to GTR and the corresponding loss according to STR as described in **H4**, for a general parabola  $v(t) = k - k \cdot t$  and  $h(t) = k \cdot t - 0.5 \cdot k \cdot t^2$  describing an object that returns after 2 seconds to its place of departure. Take the first derivative to the total gain and show that for  $k = g$  it is a maximum!
5. Bertrand Russell used for the "principle of maximum proper time" the somewhat rakish expression "principle of cosmic laziness". To what extent is that justified?
6. Obtain somehow a sphere attached to two "horns" as described in section **H5** (if necessary, simply use two funnels and two tubes). You can then use long, narrow strips of paper attached to this surface to determine the space-time paths of different objects that fall through the earth. What does the total length of the paper represent?
7. Let a light beam pass through the tunnel of the model of task 6. Do you see clearly that it takes a little longer than an observer in the OFF would expect? This is precisely the Shapiro effect, which we will calculate in **I3**.
8. Now shoot a rocket through the earth of the model of task 6. In contrast to the light beam the traversal time for the rocket is reduced thanks to gravity!
9. A sense of 'straight lines' on curved surfaces (so-called 'geodesics') can be obtained by wrapping an ace bandage around your ankle. Which path does the fabric 'itself' wish to take? (suggested by Hans Walsler)
10. Read the first three chapters of Kip S. Thorne's book "Black Holes & Time Warps - Einstein's Outrageous Legacy" [34]
11. The principle of Fermat for the path of a light beam is valid even in GTR! What does this mean for a beam of light, that passes close to the edge of the sun?
12. Enjoy the two books by Harald Fritzsch in which Newton, Einstein and a Bernese physicist of the present discuss the STR and GTR ([35] for the STR and [36] for the GTR). Newton shows his brilliant and critical mind as he is willing to learn, but first must be presented with good arguments. Einstein is repeatedly impressed how much progress experimental physics has made. But that the STR and GTR have passed all experimental tests does not surprise him one bit ...



Following the confirmation by Eddington of his predicted value of light deflection at the solar limb Einstein was transformed by the press virtually overnight into a celebrity of the first order. Occasionally, he enjoyed this role, but it was mostly just annoying to him. He has demonstrated that himself with different comments:

"To punish me for my contempt of authority, Fate has made me an authority myself." [17-9]

"In the past it never occurred to me that every casual remark of mine would be snatched up and recorded. Otherwise I would have crept further into my shell." [17-15]



The Spiral Galaxy M66 (NGC 3627)  
VLT MELIPAL/YEPUN + FORS1/FORS2

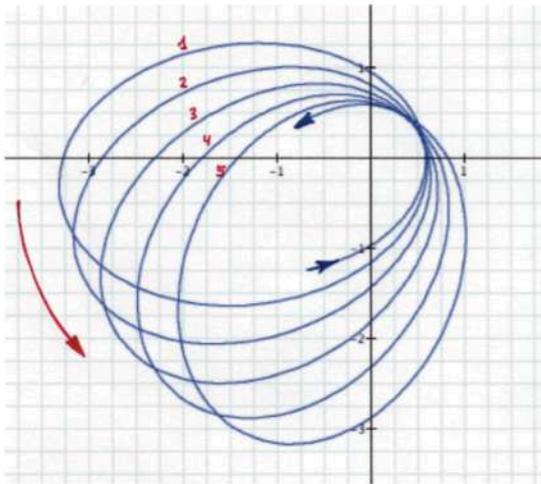
ESO PR Photo 33c/03 (19 December 2003)

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## I Testing the General Theory of Relativity

In this section we present both old and recent observations and experiments which Newton's theory can not explain but which the GTR predicts to a great precision. Some of these experiments, such as the deflection of light at the solar limb, the time delay from Shapiro or the direct measurements of the effects of time using atomic clocks, can be calculated with our current resources to a good approximation of the expected results. For the perihelion of Mercury we can at least estimate the magnitude of the effect. For other experiments we must be satisfied that at least we understand that an effect is expected.

## 11 The Precession of the Perihelion of Mercury



Should a single, isolated planet orbit the sun then it must do so according to Kepler and Newton in an exact ellipse. Newton already recognized that this is not the case in the solar system because the planets affect each other gravitationally. An exact solution to the 'three-body problem' evaded even great people like Poincaré (whose attempt at a solution deeply penetrated into the territory known today as 'chaos theory'). Today, iterative numeric methods can calculate the orbits of all the planets with high precision for a long time into the future. The apsis, the straight line through the aphelion (furthest point from sun) and perihelion (closest point to sun) of the orbit precedes very slowly under the influence

of the outer planets, in the same direction in which the planets orbit. This results in a rosette-like path, where the effect as shown in the diagram is greatly exaggerated. By the way, these numerical simulations have also shown that the solar system will remain stable even over very long periods [36-315ff].

There is, however, a small difference between the calculated values for the precession of the perihelion within Newtonian physics and those measured observationally. The following table shows the numerical values in the units 'arc seconds per century'. The fuzziness of the values can be read from the column 'difference':

Planet	Newtonian value	Observed Value	Difference	Prediction GTR
Merkur	532.08	575.19	43.11 ± 0.45	43.03
Venus	13.2	21.6	8.4 ± 4.8	8.6
Erde	1165	1170	5 ± 1.2	3.8

The difference between the calculated and the measured value, especially in the case of Mercury was so big that it demanded an explanation. The French astronomer Urbain Le Verrier, who predicted in 1845 the existence and location of the new planet Neptune based on the interference of the planet Uranus, postulated in 1859 the existence of another planet Vulcan, whose orbit was closer to the sun than Mercury's.

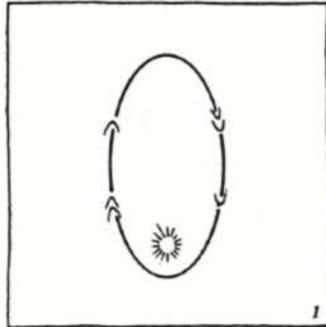
The GTR explains precisely this difference between the Newtonian theory and observation. Einstein was overjoyed when he calculated near the end of 1915 that his new theory predicted an addition of just 43 arc seconds per century to the precession of the perihelion of Mercury! He derived the following formula:

$$\Delta\phi = 3 \cdot \pi \cdot \frac{R_s}{a \cdot (1 - \varepsilon^2)}$$

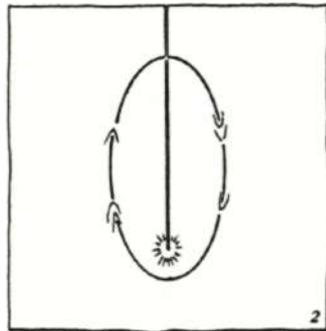
where  $\Delta\phi$  is the extra rotation per orbit in radians;  $R_s$  is the Schwarzschild radius of the sun;  $a$  is the length of the semi-major axis of the orbit; and  $\varepsilon$  is the eccentricity of the ellipse.

The effect decreases with increasing distance from the sun and is also greater with highly elliptical orbits than with circular orbits. Therefore Mercury was an ideal candidate. The small eccentricity of the orbit of Venus not only weakens the effect but also makes it difficult to observe the precession. The values in the last column of the table can be obtained from Einstein's formula, if the result is multiplied by the number of revolutions in 100 years and then converted from radians to arc seconds (See problem 1).

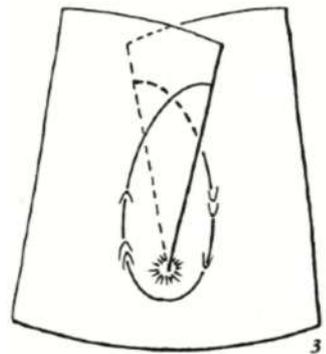
That this effect *must* occur is made nearly self-evident by Epstein's 'barrel' region [15-166]:



In the first drawing space is flat and the planet moves in its ellipse, however, with Epstein in an unconventional direction (usually one always looks from the north to the ecliptic). That is the situation according to Newton.

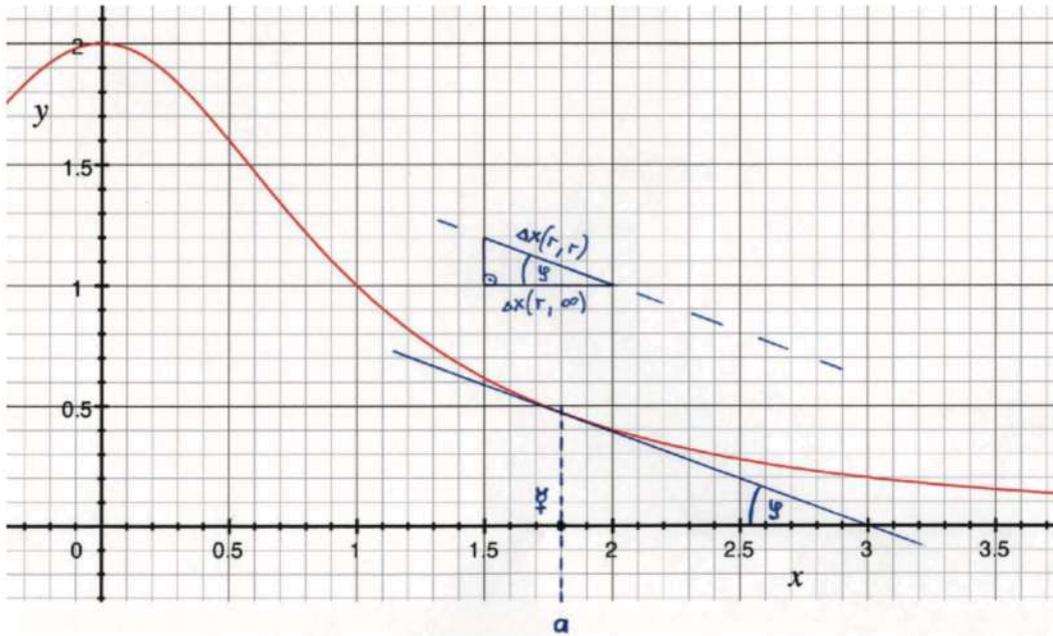


Now we cut the plane along the apsis. We make the cut from the aphelion to the sun.

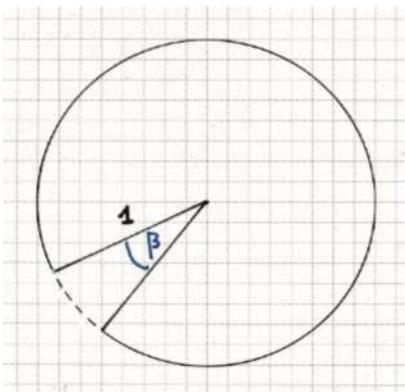


As discussed in section **H6** a cone should now arise with its tip in the sun. To this end we must push the edges on both sides of the incision over each other (thus crafts one a cone!). This forces an advance (precession) of the aphelion in the direction of the planet's orbit!

Amazingly, it is even possible to determine the magnitude of the phenomenon from Epstein's paper model. Almost with no computation we come surprisingly close to the results of the formula, whose derivation forced Einstein to tussle with elliptic integrals.



The red curve shows the cross-section of Epstein's 'space bump' (problem 5 in **I10** deals with the analysis of this function). The central mass sits at the origin, and with increasing distance  $x$  from the origin the spatial curvature decreases. If our planet has an average distance  $a$  from the central mass, we can then approximate the space bump with a local cone. The appropriate angle of inclination  $\phi$  between the surface line of the cone and the plane through the center of the central mass can be easily determined for the planet at point  $a$ . It is  $\cos(\phi) = \Delta x(r, \infty) / \Delta x(r, r) = 1 - a/a$  according to **G4**. If the cone has a surface line of length 1 then the base circle measures a radius of  $(1 - a/a)$ . We now cut the cone along a surface line and press it flat:



How big is the angle  $\beta$  of the missing sector?

$\beta / (2\pi)$  is equal to the ratio of the 'missing' arc length to the circumference, i.e.,  
 $\beta / (2\pi) = [2\pi - 2\pi \cdot (1 - a/a)] / (2\pi) = 1 - (1 - a/a) = a/a$

We thus obtain the expression for  $\beta$   
 $\beta = (2\pi) \cdot a/a = \pi \cdot 2 \cdot a/a = \pi \cdot R_s / a$

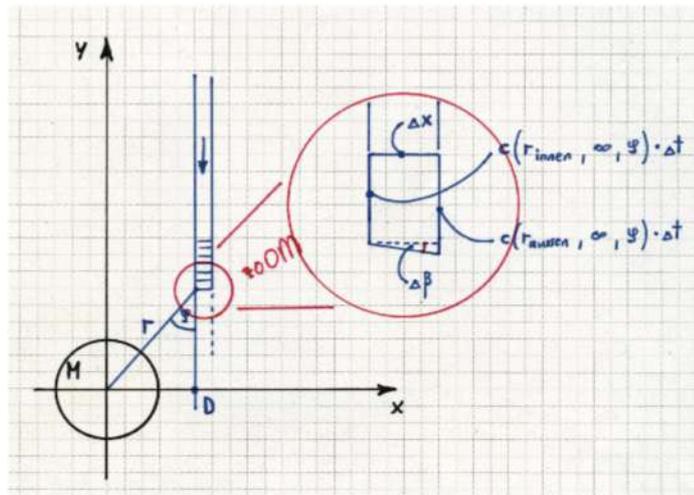
Reform the cone and you will find either a circle or an ellipse offset by about the size of this angle  $\beta$ , that is,  $\beta$  specifies the amount of precession per orbit of the apsis.

We only obtain about one third of the correct value (compare with the formula on p.134 above). This should not concern us since we have only taken the influence of space curvature into account, and that also using very modest means. We are, in any case, within a correct order of magnitude.

## 12 The Deflection of Light in the Gravitational Field of the Sun

We now want to do the calculations for the experiment, whose outcome made Einstein so famous in 1919: The deflection of light at the solar limb. The effect on the public and the clamor of the press concerning this result can only be understood against the backdrop of the just ended disaster of the First World War (see [32-232ff] or [38-191ff]). In **H6**, we already stressed that the outcome of this experiment speaks in favor of the GTR and against Newton's theory which expected only half the value predicted by the GTR.

We take the following approach: a light beam of width  $\Delta x$  passes the sun at a distance  $D$  along the  $y$ -axis. Thereby, according to our formulas in **G5**, the inner side of the beam which is closer to the sun, moves by a little bit slower than the outer side, so the wave front is tilted by a small angle  $\Delta\beta$ :



To calculate the sum of all these small changes of direction  $\Delta\beta$ , we will integrate from  $y = +\infty$  to  $y = -\infty$  using the constant value  $D$  for  $x$ . In so doing, we require that the entire change in direction is small. In principle, this is 'gravitation by refraction'!

### Step 1

We determine a workable expression for the infinitesimal change in direction  $\Delta\beta$ :

$$\Delta\beta = \tan(\Delta\beta) = \frac{[c(x + \Delta x, y, \infty, \varphi) - c(x, y, \infty, \varphi)] \cdot \Delta t}{\Delta x} = \frac{c(x + \Delta x, y, \infty, \varphi) - c(x, y, \infty, \varphi)}{\Delta x} \cdot \Delta t$$

We must integrate over the  $y$ -axis. We are not in error when we write  $dy = c_0 \cdot dt$ , even if the light beam (as seen from OFF) does not quite advance with velocity  $c_0$ . This really only means that our time slices  $dt$  are not all of the same size. Still we may overall write

$$d\beta = \frac{\partial(c(r, \infty, \varphi))}{\partial x} \cdot dt = \frac{\partial(c(r, \infty, \varphi))}{\partial x} \cdot \frac{1}{c_0} \cdot dy$$

So if we knew the partial derivative of the speed of light with respect to  $x$ , then we could write our integral:

$$\beta_{\text{total}} = \int_{+\infty}^{-\infty} d\beta \cdot dy = \int_{+\infty}^{-\infty} \frac{\partial(c(r, \infty, \varphi))}{\partial x} \cdot \frac{1}{c_0} \cdot dy$$

### Step 2

We determine the partial derivative of  $c(r, \infty, \varphi)$  with respect to  $x$ . The calculation is a bit tedious:

$$\begin{aligned} \frac{\partial}{\partial x} [c(r, \infty, \varphi)] &= \frac{\partial}{\partial x} \left[ c_0 \cdot \left( 1 - \frac{\alpha}{r} \cdot (1 + \cos^2(\varphi)) \right) \right] = && \cos(\varphi) = y/r \\ c_0 \cdot \frac{\partial}{\partial x} \left[ 1 - \frac{\alpha}{r} - \frac{\alpha}{r} \cdot \cos^2(\varphi) \right] &= c_0 \cdot (-\alpha) \cdot \frac{\partial}{\partial x} \left[ \frac{1}{r} + \frac{1}{r} \cdot \cos^2(\varphi) \right] = && r^2 = x^2 + y^2 \\ c_0 \cdot (-\alpha) \cdot \frac{\partial}{\partial x} \left[ \frac{1}{r} + \frac{y^2}{r^3} \right] &= c_0 \cdot (-\alpha) \cdot \frac{\partial}{\partial x} \left[ \frac{r^2 + y^2}{r^3} \right] = && \text{now we do derive} \\ c_0 \cdot (-\alpha) \cdot \frac{\partial}{\partial x} \left[ \frac{x^2 + y^2 + y^2}{(x^2 + y^2)^{3/2}} \right] &= c_0 \cdot (-\alpha) \cdot \frac{\partial}{\partial x} \left[ \frac{x^2 + 2 \cdot y^2}{(x^2 + y^2)^{3/2}} \right] = && \text{divide by } (x^2 + y^2)^{0.5} \\ c_0 \cdot (-\alpha) \cdot \frac{(x^2 + y^2)^{3/2} \cdot 2x - (x^2 + 2 \cdot y^2) \cdot \frac{3}{2} \cdot (x^2 + y^2)^{1/2} \cdot 2x}{(x^2 + y^2)^3} = && \\ c_0 \cdot (-\alpha) \cdot \frac{(x^2 + y^2) \cdot 2x - 3 \cdot (x^2 + 2 \cdot y^2) \cdot x}{(x^2 + y^2)^{2.5}} = && \\ c_0 \cdot (-\alpha) \cdot \frac{2x^3 + 2xy^2 - 3x^3 - 6xy^2}{(x^2 + y^2)^{2.5}} = c_0 \cdot (-\alpha) \cdot \frac{-x^3 - 4xy^2}{(x^2 + y^2)^{2.5}} = && \\ c_0 \cdot \alpha \cdot \frac{x^3 + 4xy^2}{(x^2 + y^2)^{2.5}} = c_0 \cdot \alpha \cdot \frac{x^3 + 4xy^2}{r^5} && r^2 = x^2 + y^2 \end{aligned}$$

### Step 3

We evaluate the integral:

$$\begin{aligned} \beta_{\text{total}} &= \int_{+\infty}^{-\infty} \frac{\partial(c(r, \infty, \varphi))}{\partial x} \cdot \frac{1}{c_0} \cdot dy = \int_{+\infty}^{-\infty} c_0 \cdot \alpha \cdot \frac{x^3 + 4xy^2}{(x^2 + y^2)^{2.5}} \cdot \frac{1}{c_0} \cdot dy = \\ &= \alpha \cdot \int_{+\infty}^{-\infty} \frac{x^3 + 4xy^2}{(x^2 + y^2)^{2.5}} \cdot dy = \alpha \cdot \int_{+\infty}^{-\infty} \frac{D^3 + 4Dy^2}{(D^2 + y^2)^{2.5}} \cdot dy \end{aligned}$$

On the light's path  $D = x = 2.33$  light seconds is constant. In place of  $+\infty$  to  $-\infty$  one could also integrate from 2000 to -2000 light seconds (the earth is about 500 light seconds from the sun). If so then note that  $\alpha$  must also be in these units:  $\alpha = G \cdot M / c^2 = G' \cdot M \approx 4.9261 \cdot 10^{-6}$  light seconds !

Using the TI-89 calculator to do the integration delivers a result of  $8.4571 \cdot 10^{-6}$ . This is the whole  $\beta$  deflection in radians. Converting to arc seconds, we get  $8.4571 \cdot 10^{-6} \cdot 180 \cdot 3600 / \pi \approx 1.74$  arc seconds. Using Mathematica ® with twice the precision gives the same value, namely 1.7518 arc seconds.

For our integral, [27-143] gives the anti-derivative  $(y/r + (y/r)^3) / D$ . This can easily be checked by differentiating! Now the limit of  $y/r$  as  $y$  approaches infinity is simply 1. Thus for the total deflection in radians one obtains the simple formula

$$\beta_{\text{total}} = \frac{4 \cdot \alpha}{D} = \frac{4 \cdot G \cdot M}{c^2 \cdot D} = \frac{2 \cdot R_s}{D}$$

What experimental data are available to test this formula?

[27-145] presents the analysis of photographs of star fields during solar eclipses up till 1952. The values lie between 1.61" and 2.01" with uncertainties of 0.10" to 0.45" (I have omitted so-called 'outliers'). This is enough to give GTR preference over Newton's theory, but the uncertainty is greater than 10%. The 'phone book' [29-1105] gives data from measurements with radio telescopes. Every year on October 8, the sun – as seen from the earth - moves over the quasar 3C279. Another quasar 3C273 is in the vicinity and allows a precise measurement of the angle between these two objects. In 1970 the GTR could be confirmed to within 5% accuracy. Measurements using VLBI (very long baseline interferometry) could confirm GTR in 1995 to an accuracy of  $0.9996 \pm 0.0017$ , i.e., to 1.7 parts per thousand. In 1999 an analysis of 2 million VLBI measurements was published, which delivered an accuracy of  $0.99992 \pm 0.00014$ . This data were taken from the following website on December 26, 2006:

<http://relativity.livingreviews.org/open?pubNo=lrr-2001-4&page=node10.html>

The page <http://relativity.livingreviews.org> due mainly to Clifford M. Will has rendered outstanding service and provides the most comprehensive and current information about ongoing research and experimental trials in the field of GTR.

Another confirmation of GTR comes from the data of ESO's Hipparcos satellite. This had the task of precisely measuring the position of 118,000 stars (up to size class 12.5), so that they could later be used as reference. Hipparcos (after a very inauspicious start, see Wikipedia) performed this task brilliantly. The angular resolution of the position measuring instruments was 0.001 arc seconds. Thus, it could confirm the predictions of GTR across the entire sky with an accuracy of about 0.3%. Light deflection can already be measured, if the light moves past the sun at a distance of one astronomical unit, i.e., a distance of 150 million kilometers! When we direct our sight from the earth along the y-axis, i.e., in a direction perpendicular to the segment connecting the earth to sun, then the light beam has already suffered exactly half of the total  $\beta$  deflection.  $\beta = 2 \cdot R_s / D$  with  $D = 1$  AU has the value 0.0081 arc seconds, and half is still 4 thousandths of an arc second, i.e., four times the accuracy of the Hipparcos satellite!

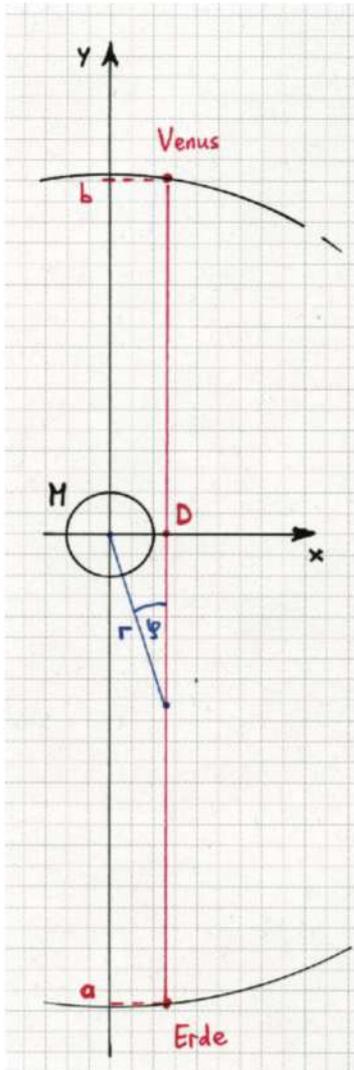


"He denies the Big bang Theory !"

Oswald Huber, Neue Zürcher Zeitung, Sunday Edition, November 12, 2006  
© Oswald Huber & NZZ

### 13 The Shapiro Experiment

In I2 we saw that a beam of light that passes close to the sun barely changes its direction. On the other hand, the time needed for the traversal of a given path diameter grows significantly, because the light, in accordance with the formulas of G5, moves more slowly than it would when outside a gravitational field. Problem 5 of H7 illustrates the effect of spatial curvature, but a virtually equal proportion is also due to the curvature of space-time. It is again an effect that can be well understood viewed as 'gravitation by refraction'.



In 1962 Irwin Shapiro suggested that this delay be measured by sending some strong radio signals to Venus, when it is in opposition to the Earth, and then measuring the time it takes for the (extremely weak) reflected signals to arrive.

When in 1964 the 120 foot Haystack antenna in Westford, US was left by the military to MIT, Shapiro and his team began plans to carry out the experiment. The experiment first took place from November 1966 until August 1967. "It would have been nice to prove Einstein wrong," said Shapiro later. That has not been granted him since all experiments up till 2006 have confirmed the GTR within the specified accuracy.

Shapiro lowered the imprecision of his initial measurements from over 3% to less than 1% in subsequent years. Newer versions of this experiment work with transponders on space probes. These receive the signal from the earth and after a precisely known delay send it with increased intensity back to earth. Thus with the Viking Mars probe of 1979 the predictions of the GTR for this delay in the gravitational field of the sun could be confirmed to an accuracy of 0.1%. In 2003, with the space probe Cassini an accuracy of 0.0012% was achieved!

In the situation presented on the left an observer in OFF would measure values of  $a = -498.67$  and  $b = 370.70$  (as in I2, we calculate everything in units of light seconds, so that  $c_0 = 1$  and  $\alpha \approx 4.9261 \cdot 10^{-6}$ ). Without gravity one would expect a duration of  $2 \cdot (b - a) / c_0 \approx 2 \cdot (370.70 + 498.67) / 1 \approx 1738.74$  seconds. In the following we calculate the difference in time that arises because the light near the sun travels a little bit slower.

With gravity the duration in both directions (with  $c_0 = 1$  !) is

$$T = 2 \cdot \int_a^b \frac{1}{c(r, \infty, \varphi)} \cdot dy = 2 \cdot \int_a^b \frac{1}{c_0 \cdot (1 - (1 + \cos^2(\varphi)) \cdot \alpha / r)} \cdot dy = 2 \cdot \int_a^b \frac{1}{1 - (1 + \cos^2(\varphi)) \cdot \alpha / r} \cdot dy$$

This integral is numerically very unstable. A substitution using  $1/(1-x) = (1+x)/(1-x^2)$  helps since we then eliminate in the denominator the very small  $x^2$  term:

$$T = 2 \cdot \int_a^b \frac{1 + (1 + \cos^2(\varphi)) \cdot \alpha / r}{1 - ((1 + \cos^2(\varphi)) \cdot \alpha / r)^2} \cdot dy = 2 \cdot \int_a^b 1 + (1 + \cos^2(\varphi)) \cdot \alpha / r \cdot dy = 2 \cdot \int_a^b 1 + \frac{\alpha \cdot (1 + \cos^2(\varphi))}{r} \cdot dy$$

This is the entire time there and back with gravity. The difference to the expected value without gravity is

$$\Delta T = 2 \cdot \int_a^b \frac{\alpha \cdot (1 + \cos^2(\varphi))}{r} \cdot dy = 2 \cdot \alpha \cdot \int_a^b \frac{(1 + y^2 / r^2)}{r} \cdot dy = 2 \cdot \alpha \cdot \left[ \int_a^b \frac{1}{r} \cdot dy + \int_a^b \frac{y^2}{r^3} \cdot dy \right]$$

This integral can be evaluated both numerically and analytically. For  $r$  the root of  $D^2 + y^2$  is used. For a path that passes directly the solar limb,  $D \approx 2.33$ ; and using  $a = -499$  and  $b = 371$  the calculator delivers for the total 'delay' of the signal the value  $\Delta T \approx 213.3 \mu s \approx 0.000,213,3$  seconds - an easily measurable value.

With help of an integral table, or a computer algebra system one can find an anti-derivative:

$$\int \frac{1}{\sqrt{D^2 + y^2}} \cdot dy + \int \frac{y^2}{\sqrt{D^2 + y^2}^3} \cdot dy = 2 \cdot \ln(y + \sqrt{D^2 + y^2}) - \frac{y}{\sqrt{D^2 + y^2}}$$

Setting the limits of integration and using additional symbols

$a_V \sim$  Sun-Venus distance ,  $y_V \sim$  y-coordinate of Venus ,  $y_V > 0$

$a_E \sim$  Sun-Earth distance ,  $y_E \sim$  y-coordinate of Earth ,  $y_E < 0$

$\varphi_V \sim$  angle Sun-Venus-Earth

$\varphi_E \sim$  angle Sun-Earth-Venus

we get the following expression for the total amount of delay which also provides good values for a path at a great distance from the sun:

$$\begin{aligned} \Delta T &= 2 \cdot \alpha \cdot \left[ 2 \cdot \ln \left( \frac{a_V + y_V}{a_E + y_E} \right) + \frac{y_E}{a_E} - \frac{y_V}{a_V} \right] = 2 \cdot \alpha \cdot \left[ 2 \cdot \ln \left( \frac{(a_V + y_V) \cdot (a_E - y_E)}{a_E^2 - y_E^2} \right) + \frac{y_E}{a_E} - \frac{y_V}{a_V} \right] = \\ &= 2 \cdot \alpha \cdot \left[ 2 \cdot \ln \left( \frac{a_V \cdot (1 + \cos(\varphi_V)) \cdot a_E \cdot (1 + \cos(\varphi_E))}{D^2} \right) - \cos(\varphi_E) - \cos(\varphi_V) \right] \end{aligned}$$



The 120 foot radio antenna at MIT in Westford / USA with which Shapiro in 1966/67 carried out his first experiment.

In opposition the two angles  $\varphi_E$  and  $\varphi_V$  are very small and we may set the cosine value to 1. For this special situation this gives us the simpler formula

$$\Delta T = 2 \cdot \alpha \cdot \left[ 2 \cdot \ln \left( \frac{a_V \cdot 2 \cdot a_E \cdot 2}{D^2} \right) - 1 - 1 \right] = 2 \cdot \alpha \cdot \left[ 2 \cdot \ln \left( \frac{4 \cdot a_V \cdot a_E}{D^2} \right) - 2 \cdot \ln(e) \right] = 2 \cdot \alpha \cdot \left[ 2 \cdot \ln \left( \frac{4 \cdot a_V \cdot a_E}{e \cdot D^2} \right) \right]$$

$$\text{also } \Delta T = 4 \cdot \alpha \cdot \ln \left( \frac{4 \cdot a_V \cdot a_E}{e \cdot D^2} \right)$$

Even this simple formula provides for  $D = 2.33$  (solar limb),  $a_E = -499$  and  $a_V = 371$  a delay of  $213.3 \mu\text{s}$ .

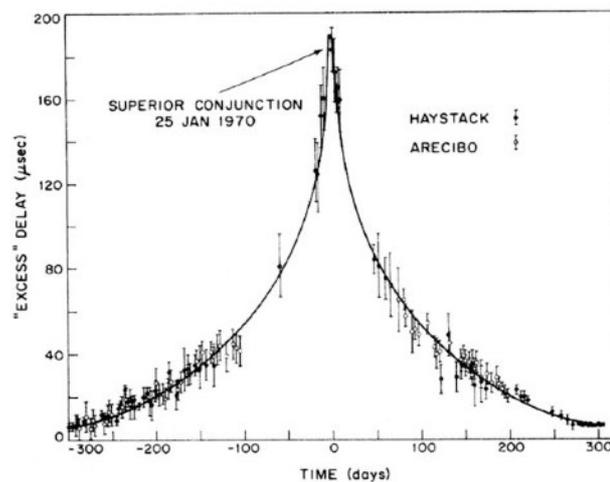
Where are the flaws of these calculations ?

The first simplification concerns the path of integration. Light follows a geodesic and not a straight line. The angle of deviation is small (about  $1.74''$ ) but the length of the path might be influenced. In [29-1106] Misner et al. pretend that the first ten digits are not affected from this simplification.

Second, the time delay is calculated for the observer in the OFF. So we have to correct for the STR-effect from the earth's orbital speed and for the GTR-effects of the gravitational fields of the sun and the earth (cf. problem 13 in **G6** !). Doing these calculations gives us a correction factor of  $0.999'999'984'5$  to adapt the time delay to earth-bound measurement. So we can ignore that, too.

However, the third flaw *has* some influence. In what type of coordinate systems do we get the distances from our astronomy programs? We did the calculations in Schwarzschild metrics. My astronomy program probably does all its calculations in Euclidean metrics in a Newtonian world. Wheeler gives in [29-1106] a simple derivation in PPN-coordinates, leading to the same formula as ours, but without Euler's  $e$  in the denominator.

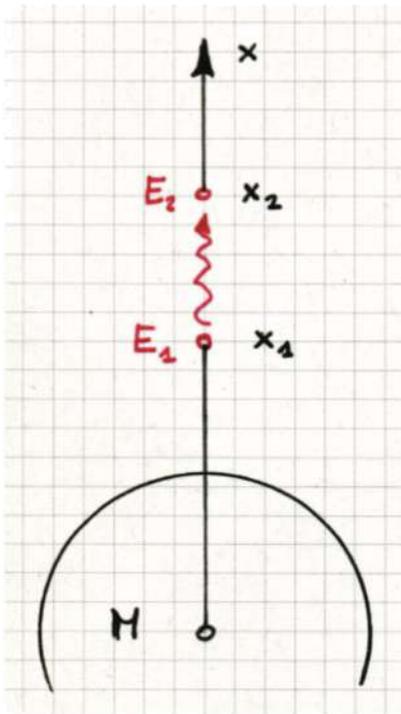
Let us therefore confront the theory with experimental data. For the Venus opposition of 1970-01-25 my astronomy program "Starry Night Pro" yields the values  $a_E \approx 491$ ,  $a_V \approx 363$  and  $D \approx 9.42$ . Plugging these values into our formula we get a delay of  $157 \mu\text{s}$ . Without the  $e$  in the denominator we get  $177 \mu\text{s}$ , coming very close to Shapiro's measurement. However, not the delay is measured, but the total signal runtime! To get an expectation of a "zero-delay" you have to start measurements months before the day of opposition. The delay then is *calculated* from the distance Venus and Earth should have if they follow their Keplerian ellipses.



The Shapiro effect is also interesting because it decreases only slowly with increasing distance  $D$  from the central mass. The light deflection following the formula in **I2** is proportional to  $1 / D$ . The Shapiro delay, however, is essentially proportional to  $1 / \ln(D)$ , as is seen from the formula above. At a distance of 100 solar radii the value of the deflection of light decreases to 1%, but the delay there is still 21% of the maximum effect at the solar limb. One speaks, therefore, of a 'long-range effect'.

## 14 The Experiment of Rebka and Pound

We have already seen in **G4** that Schwarzschild attempted beginning in 1913 to demonstrate a 'red-shift' in the absorption lines of the sun's spectrum. With this Einstein had hoped to obtain a first experimental confirmation of his theory. However, the evidence would not have specifically confirmed the GTR but rather the conservation of energy:



As a photon flies from  $x_1$  to  $x_2$  its potential energy increases and it therefore needs to release a bit of its internal energy  $E_1 = h \cdot f_1$  (where  $h$  is Planck's constant). At location  $x_2$  it has the smaller energy  $E_2 = h \cdot f_2$ , the frequency of the radiation is thus slightly smaller and the wavelength (due to the formula  $c = f \cdot \lambda$ ) becomes somewhat bigger. The wavelength thus shifts in the direction of the red end of the optical spectrum, from whence the name 'red-shift' comes. For a small rise of the photon from  $x_1$  to  $x_2$  applies

$$\begin{aligned} \Delta E_{\text{pot}} &= \frac{E_1}{c^2} \cdot G \cdot M \cdot \left( \frac{1}{x_2} - \frac{1}{x_1} \right) = h \cdot f_1 \cdot \frac{G \cdot M}{c^2} \cdot \left( \frac{1}{x_2} - \frac{1}{x_1} \right) \\ &= h \cdot \alpha \cdot f_1 \cdot \left( \frac{1}{x_2} - \frac{1}{x_1} \right) = h \cdot \alpha \cdot f_1 \cdot \left( \frac{x_1 - x_2}{x_2 \cdot x_1} \right) \end{aligned}$$

Putting this together we get

$$\begin{aligned} h \cdot (f_1 - f_2) &= h \cdot \alpha \cdot f_1 \cdot \left( \frac{x_1 - x_2}{x_2 \cdot x_1} \right) \\ \text{oder } \frac{\Delta f}{f} &= \alpha \cdot \left( \frac{x_1 - x_2}{x_2 \cdot x_1} \right) = \frac{G \cdot M}{c^2} \cdot \left( \frac{x_1 - x_2}{x_2 \cdot x_1} \right) \end{aligned}$$

For very small uplifts near the earth's surface this can be further simplified:  $x_1 \cdot x_2 \approx r_E^2$ , for the difference  $x_1 - x_2$  we write  $\Delta x$  and the term  $G \cdot M / r_E^2$  is simply the gravitational acceleration  $g$  at the earth's surface. Thus we get the formula

$$\frac{\Delta f}{f} = \frac{G \cdot M}{c^2} \cdot \left( \frac{\Delta x}{r_E^2} \right) = \frac{g \cdot \Delta x}{c^2}$$

This result – albeit with a different justification – was already derived in **G4**! There we also calculated that at a height difference  $\Delta x$  of 22.6 meters we have a ratio  $\Delta f / f$  in the range of  $10^{-15}$ .

The two American physicists R.V. Pound and G.A. Rebka succeeded in 1960 in experimentally measuring this tiny effect with an accuracy of about 10%. In 1964 Pound and J.L. Snider increased the accuracy to 1%. They used the extremely sharp spectral lines of radioactive cobalt atoms, which due to their embedding in a crystal lattice of iron atoms are emitting and absorbing virtually recoil-free (keyword Mössbauer effect). The height difference of 22.6 meters was enough to put the source and absorber out of resonance. Then using a screw thread the absorber was moved so 'fast' (a few millimeters per hour, see problem 9 in **I10**) in the direction of the source until the Doppler effect again put it in resonance with the source. The speed needed for the maximum resonance was then measured to calculate the frequency shift  $\Delta f$ !

## 15 Hafele and Keating Travel Around the World

Around 1960 the accuracy of cesium atomic clocks was so great that one could consider directly verifying the effects of STR and GTR with such clocks. While other experiments using satellites were being planned, J. Hafele and R. Keating quietly prepared a test using ordinary commercial airliners. In October 1971 they flew - first in an eastward direction and then in a westward direction - around the world. Together with the 4 atomic clocks they occupied 4 first-class seats. During the flight they continuously recorded their altitude, speed and direction.

GTR requires clocks at higher altitudes to run faster than identical clocks on the ground. STR requires that more time elapse for a clock at rest than for one moving. One must, however, be in an inertial frame which the rotating earth is not! The clocks on the ground have as a result of the earth's rotation a significant speed (given that they are not at the North Pole ...). This speed must be added to the speed of the aircraft during the eastward flight. When flying westward it must, however, be subtracted, which after extensive analysis leads to the following table of expected time differences compared to the clocks 'left behind':

Predicted Effect	Flying East	Flying West
GTR (Gravitation)	+ 144 ± 14 ns	+ 179 ± 18 ns
STR (Velocity)	- 184 ± 18 ns	+ 96 ± 18 ns
Total	- 40 ± 23 ns	+ 275 ± 21 ns

The +273 ± 7 ns measured by Hafele and Keating on the westward flight are in (almost suspiciously) good agreement with the expected value. On the eastward flight the clock with serial number 361 and its -74 ns seemed to be marching to its own drummer, but the other three with -51 to -57 ns were in harmony. The average of all four clocks was - 59 ± 10 ns.

Particularly nice aspects of this experiment are, firstly, that the effects of the STR and GTR can in a certain sense be separated, although a given clock is always measuring the total impact. Secondly, it is simply cool how these two with minimalistic resources stole the show from the others with their expensive satellite experiments.

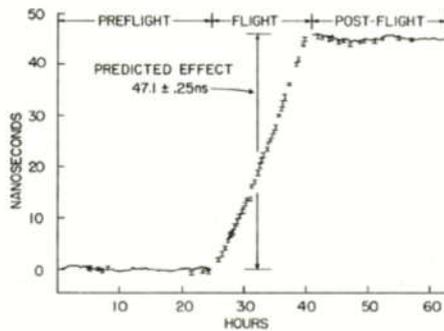
Hafele and Keating could confirm the predictions of the GTR and STR with an accuracy of about 9%. A significant increase was achieved in the experiment of Maryland, which we discuss in 16. With a rocket flight with a hydrogen powered maser on board which served as the clock, R.F.C. Vessot and M.W. Levine finally reached in 1979 after several years of data analysis (!!) a result with an uncertainty of ± 0.02% which is within the limits of the predictions of the STR and GTR. This rocket is now flying under the title 'Gravity Probe A' (18). Today these clock experiments are performed, so to speak, in the opposite direction: We assume that the atomic clocks in the GPS satellites are 'wrong' according to STR and GTR, correct the time signals accordingly, and generate the 3D position near the earth's surface to within a few centimeters based on precise orbital data and the corrected signals from four GPS satellites (see 17).

Addendum May 2009: My skepticism concerning the too-good-to-believe match between prediction and experimental results seems to be confirmed. A.G. Kelly points out in an internet post that the recently published original data of Hafele and Keating in no way allow a serious derivation of the results which made the two famous. Instead of a 'cool' or 'clever' experiment we should perhaps rather speak of fraud. The link: <http://www.cartesio-episteme.net/H&KPaper.htm> (september 2009)

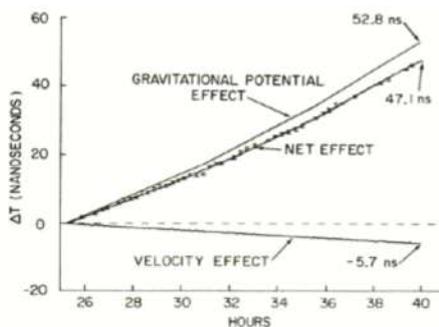
## 16 The Maryland Experiment

A group of researchers from the University of Maryland (USA) carried out the Hafele and Keating experiment in a more scientific manner. Instead of ordinary commercial aircraft flying imprecise routes, they used an aircraft of the U.S. Navy whose route was very carefully tracked during the whole flight. This plane could complete its circuit in 15 hours flying at a height of up to 35,000 feet (~10,500 m). In addition, during the flight the time of the three atomic clocks on board was continually compared with that of the three clocks of the same type on the ground by exchanging laser pulses with a pulse width of 0.1 ns. The three clocks were carefully shielded against vibration, temperature variations, pressure variations and the influences of magnetic fields. The differences due to construction between the 6 clocks used in the experiment were accurately measured, before, between and after the flights, and their values were corrected according to these measurements.

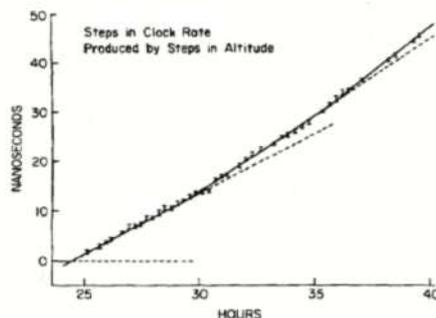
After several test flights, five main flights, each lasting 15 hours were flown and analyzed. All six (corrected) clocks ran before, between and after these flights at the same speed, however, during the test flights they accumulated a difference, which exactly corresponded to the predictions of STR and GTR. The following plots are from the second of the five flights, which took place on November 22, 1975:



The second graph actually shows the same data as the first one but with an expanded time scale and also including the calculated influences of the STR and GTR. Since the flight speed (except for the relatively short periods during take-off and landing) was largely held constant (and as small as possible!), the STR 'velocity effect' shows itself as practically linear.



Including the influence of the gravitational potential one clearly sees "kinks" in the graph on the left. They arise from the fact that the aircraft had to first fly 5 hours at 25,000 feet before it had used enough fuel to allow it to fly at the next altitude of 30,000 feet. After a further 5 hours, it was light enough to climb to the targeted maximum altitude of 35,000 feet. These 'altitude levels' are particularly emphasized in the third graph.



All in all C.O. Alley and his team were able to confirm the predictions of STR and GTR to an accuracy of about 1.6%. A nice summary report of this experiment was written by Alley himself [39]. Our summary and the graphs were taken from that report.

## 17 GPS, LRS and Relativity

Atomic clocks, following their introduction in the 50s, quickly became more accurate and smaller. Thus, in 2003, a rubidium atomic clock was successfully built occupying a volume of 40 cm<sup>3</sup>, consuming 1 Watt of power and having an accuracy of  $3 \cdot 10^{-12}$  ! Pierre Thomann of the Observatory of Neuchâtel and Gregor Duddle of the Federal Office for Metrology (METAS) in Berne also succeeded in 2003 in increasing the accuracy of the standard cesium clock using a special cooling technique by a factor of 40 to  $1 \cdot 10^{-15}$ .

Starting in 1958 the U.S. military began to combine these clocks with other advances in electronics and satellite technology into a worldwide global positioning system (GPS). The first working system, TRANSIT, was deployed in 1964 and primarily had the function of guiding submarine missiles to their destination. Better well-known was NAVSTAR, which was formally put into operation on July 17th, 1995 and was also available for civilian use. 24 satellites circle the earth in well-known orbits twice a day and continually transmit time signals. Small and cheap receivers can use the tiny time differences of the signals from at least 4 of these satellites to determine their own position to a few meters and also the time (four measurements determine the four unknowns). Recall: 1 nanosecond corresponds to a distance of 30 centimeters. If the receiver itself had a highly accurate and perfectly synchronized clock then two or three satellites would be sufficient to determine its position with an accuracy of a few centimeters. The uncertainty would then be mainly from the imprecise knowledge of the trajectory of the satellites.

A small group at the University of Bern is working at the forefront of this problem: In Zimmerwald laser pulses are sent to reflectors attached to the satellites specifically for this purpose. The orbit of the satellites can be precisely determined to within centimeters from the few photons of the reflected signal that can be captured. The precise orbital data allow the off-line evaluation of the GPS satellite signals with special software from the University of Bern achieving accuracy sufficient for surveying purposes. In this way one can today directly measure the folding of the Alps, the drift of continents or the earth tides. LRS (Laser Ranging Systems) are also available in Germany, in both Potsdam and Wetzell in Bavaria. All these stations are operating in a world wide collaboration: <http://ilrs.gsfc.nasa.gov/> .

The former Soviet Union has also built a military-controlled satellite navigation system (GLONASS). The European Space Agency (ESA) is preparing its own, civilian-controlled system (GALILEO). The first satellites are already in space (the launch could not be delayed because otherwise ESA would have lost a reserved frequency band ...). In addition to the ESA member states, China, India, Canada and Israel are also participating on this project. The clocks being used for the Galileo satellites have been developed by the Neuchâtel Institute mentioned above.

All of these satellite-based navigation systems would never work without taking into account the STR and GTR. The corrections for the STR, as well as those of the GTR are not even constant over an entire orbit since the orbits of the satellites are always slightly elliptical. These variations in both altitude and relative speed must be taken into account for high precision position measurements (to within a few millimeters). The influences are exactly those that we discussed in **I5** and **I6**.

Since these Global Positioning Systems have arisen in the www-age, you will find a wealth of descriptions and illustrations in the web. Also the indispensable 'counterpart', the LRS, is well documented in the web, although it is much less well-known to the public.



The inconspicuous facility of the Astronomical Institute of the University of Berne in Zimmerwald.

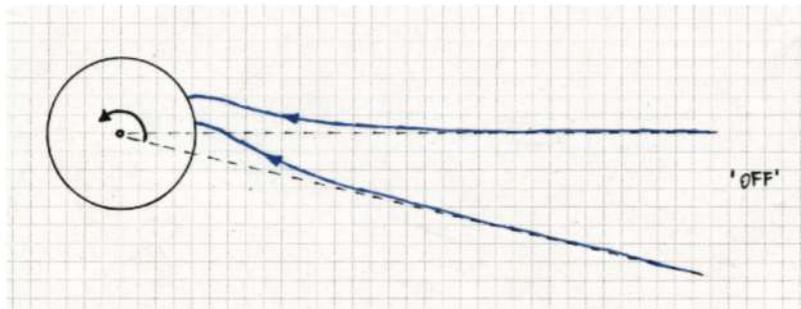


The heart of the facility at Zimmerwald: A laser which delivers ten very intense and sharp pulses per second and which are then sent to the reflector on the satellite through a small telescope (see picture above, domed building on the left). To accomplish this, the approximate position of the satellite must, of course, already be known. The few photons that arrive within the narrow time window and also have the correct wavelength will be recognized as a reflected signal and used to determine the round-trip time with a precision in the range of 0.1 nanoseconds.

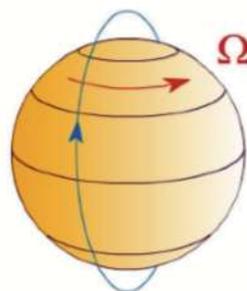
Both photos on this page are snapshots taken by the author.

## 18 The Einstein-Thirring-Lense Effect

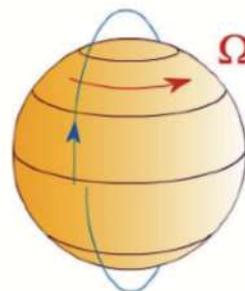
Newton's experiment with the hanging water bucket seems to show that the water in the bucket knows if it is rotating or not. Ernst Mach criticized Newton's conclusion in 1883 that absolute space exists and asserted that a rotating mass must influence an inertial frame in its vicinity. The two Austrian physicists Hans Thirring and Josef Lense showed in 1918 that Einstein's GTR solves this problem: A rotating mass slightly drags the metric of space-time along with it, twisting it a little or in the extreme case, creating a vortex-like structure in space-time. A free-falling object from the OFF no longer moves, therefore, in a 'straight' line toward the center of a spherical central mass if it is rotating:



This 'dragging' of space (keyword 'frame dragging') must be noticeable for a satellite orbiting the earth around the poles through a small rotation of its orbital plane:



according to Newton



according to Einstein-Thirring-Lense

Prof. Franz Embacher has kindly granted use of this graphic from his presentation on the Lense-Thirring effect on <http://homepage.univie.ac.at/Franz.Embacher/Rel/>. This website is really a true treasure!

In October 2004, I. Ciufolini and E.C. Pavlis of the University of Lecce presented their analysis [40] of the orbital data of two satellites (LAGEOS and LAGEOS 2). These satellites were specifically intended as targets for Laser Ranging (see 17). From the fluctuations and irregularities in their orbits geologists have gained much information on the detailed structure of earth's gravitational field as well as the density distribution in the earth's interior. Ciufolini and Pavlis have, with great effort, eliminated mathematically all other influences on the orbits of these satellites (for example, the radiation pressure of the sun!) in order to finally isolate the tiny (31 milli-arcseconds per year) Lense-Thirring effect. They believe they have succeeded to an accuracy of approximately  $\pm 10\%$ .

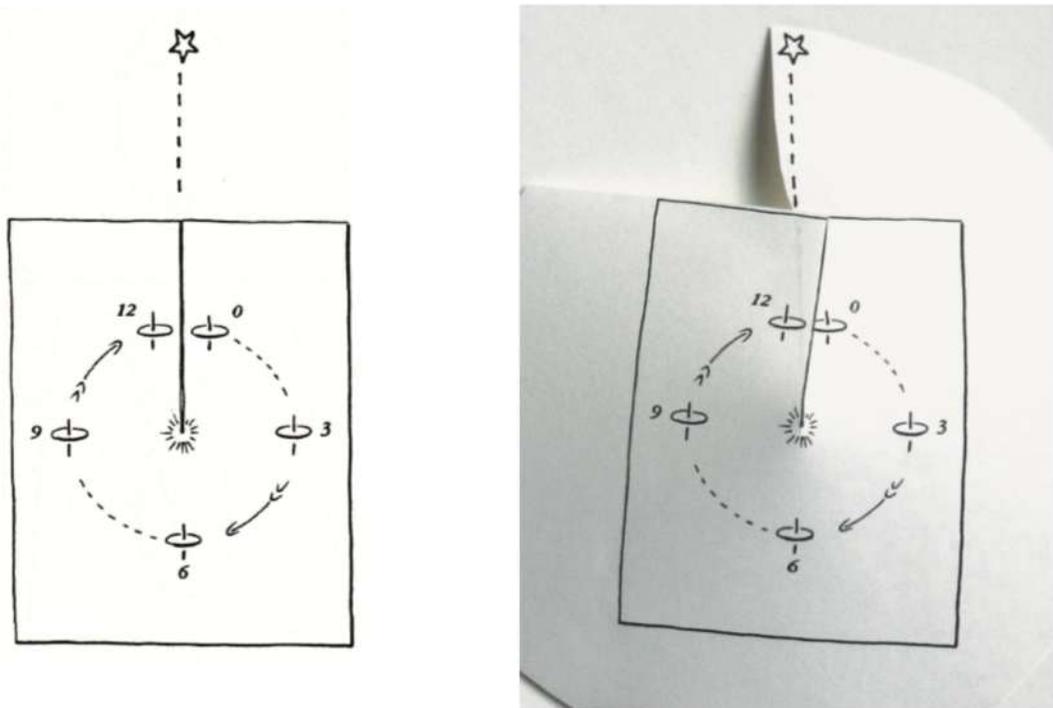
Scientists from NASA and Stanford University want to measure this effect with 1% accuracy with the satellite 'Gravity Probe B' using a different method. You can find detailed information on the Internet - it is fascinating how this experiment once again tests the limits of what is technically feasible! The actual measurements were completed in August 2006 and the evaluations should now (January 2007) be complete. After a critical review by experts, the outcome will be presented in April 2007. Just after the calculations of Ciufolini and Pavlis nobody expects that the GTR will be refuted. However, if Einstein's GTR can be confirmed at the 1% level, it will mean the end of some competing theories.

Gravity Probe B uses four gyroscopes constructed from high-precision polished quartz spheres. The changes in the gyro's axis relative to the satellite are determined. This is aligned, using a small telescope, as stably as possible to the star HR 8703 in the constellation Pegasus, whose motion is very well known. The following two effects are simultaneously measured:

1. The 'geodetic' effect, which we met in **I1** as the precession of the perihelion of Mercury and consists of a tilting of the gyro axis in the orbital plane. It should be about 6.6 arc seconds. The accuracy of Gravity Probe B should be better than 0.01%.
2. The Lense-Thirring effect, which consists of a tilting of the gyro axis perpendicular to the orbital plane. It should be about 0.041 arc seconds. This effect could be measured to an accuracy of about 1%.

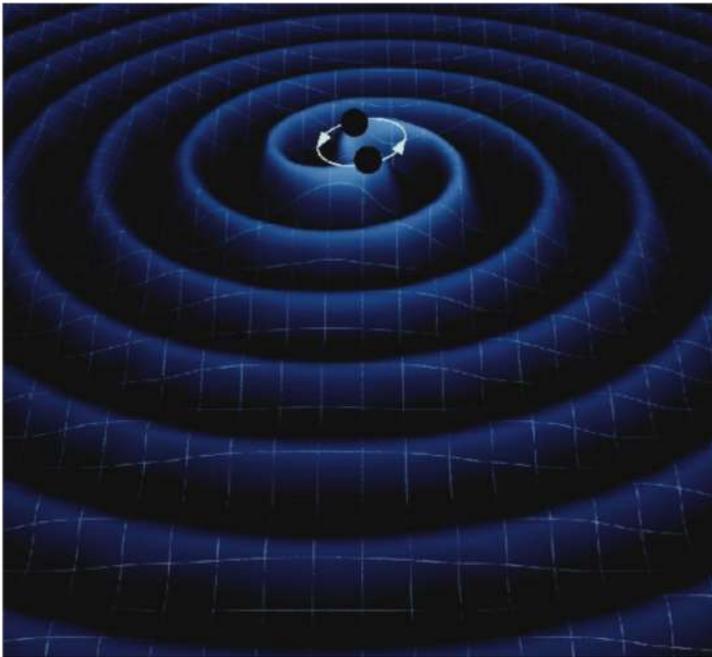
I would encourage you once again to have a look in the Internet at the reports, pictures and other material prepared for this latest test of the GTR. Use the search keywords mentioned above, as they are longer alive as actual concrete addresses. Addendum of January 2009: The experiment was not able to attain its lofty goal. The signal due to a 'frame dragging effect' is drowned in an unpredicted 'noise'.

The geodetic effect can be nicely illustrated with Epstein [15-177f]. We simply repeat the idea of Section **I1** and draw an additional rotational axis. On the left we have Newton's 'flat' world, in which the gyro maintains its direction absolutely and on the right in the curved space of GTR the gyro is tilted through a small angle after one revolution:



## 19 Gravitational Waves

The *immediate* effect of the change of a gravitational field on distant objects is a particular problem of Newton's gravitational theory! If we quickly move a mass a few meters then the corresponding effect is *immediately* felt throughout the entire Milky Way. Time does not appear at all in Newton's force law! If we move a mass periodically back and forth, it creates an immediate force at arbitrary distances, that cause a small sample mass to vibrate. Any change in a gravitational field imparts an instantaneous transmission of energy and information across arbitrary distances! However, the STR sets with the speed of light an upper limit for any information transfer.



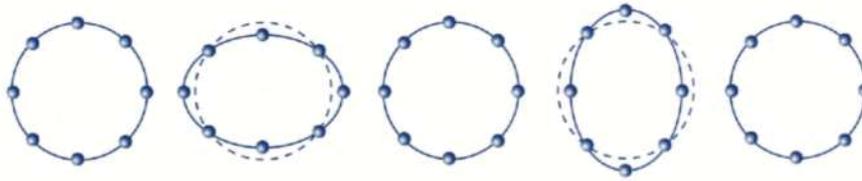
Einstein showed in 1918 that a change in the distribution of mass and therefore its impact on the structure of space-time propagates according to GTR at the speed of light. Thus two massive objects which are rapidly circling each other should emit gravitational waves which are transmitted at the speed of light throughout space.

This visualization of gravitational waves was taken from <http://lisa.jpl.nasa.gov/gallery/>

However, the effects on the structure of space-time, even from the most extreme sources, are extremely small. In 1987 in the Large Magellanic Cloud, a small companion galaxy of our Milky Way, a supernova explosion was observed (the explosion had taken place 160,000 years earlier ...). The 'gravitational bolt' on earth was a hundred times more intense than the radiation of the sun on the earth. However, it caused the distance from earth to sun to 'fluctuate' by only a few atomic diameters!

The length changes in the much shorter 'arms' of the existing detectors of such signals (300 m for TAMA300, 600 m for GEO600, 3 km for VIRGO and 4 km for LIGO) are correspondingly smaller. It has not yet been demonstrated that these facilities are capable of detecting these effects at all. Since early 2006 scientists are waiting for the first signals. One is also dependent on the cooperation of several detectors: due to the enormous background noise, one can be reasonably certain a signal has been found, only when it has simultaneously been registered by multiple, widely separated detectors.

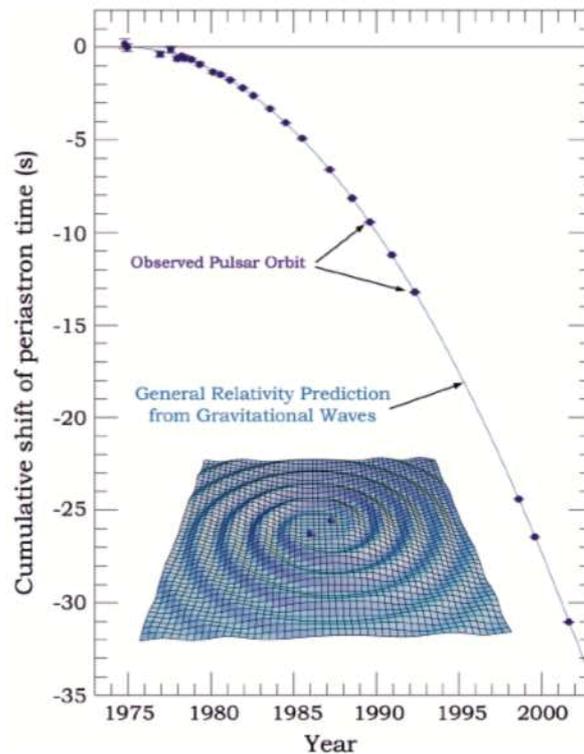
Satellite-based projects such as LISA have better chances of detecting a signal since the receiver is not exposed to terrestrial interference and since the arm length of the detectors can be tens of thousands of kilometers. More information can be found at the corresponding web-sites of NASA and ESA.



All ground based detectors have two arms perpendicular to each other. Gravitational waves are quadripole: If such a wave strikes perpendicular to the plane it affects 8 circularly arranged free falling test masses as shown above. First, the space in one direction is compressed and in the direction perpendicular to it expanded. Then the same happens in opposite directions. The metric of space-time 'billows' a little bit. This means that the elapsed time in the split halves of the laser beam in the two arms of the interferometer fluctuate for a short time at *fixed* mirrors. Therefore, when the two halves of the split beam are reunited the interference results no longer in permanent darkness (the default), but rather a brief flare becomes visible.

It is quite possible that tomorrow, for the first time, a gravitational wave will be clearly and directly demonstrated. This will require a bit of luck since supernova explosions, even in a large galaxy such as ours, do not occur every day. The last two were discovered by Tycho Brahe (1574) and Johannes Kepler (1604) – thus ringing the death knell on the idea of the immutability of the celestial sphere of fixed stars.

However, we have already had for a long time a very accurate indirect confirmation of Einstein's gravitational waves. For more than 30 years, astronomers have measured the star system B1913+16. Two neutron stars of about 1.5 solar masses and a diameter of 20 km (!) orbit each other as shown in the picture above. A rotation takes less than 8 hours. Since this system radiates energy in the form of gravitational waves the two components must continually come closer. Thus, the orbital period gets shorter and shorter. This rotation can be measured very precisely, because the radio cone of one of the two stars passes the earth once each rotation: It is a pulsar. The decrease in the orbital period of 0.000,076,5 seconds per year, agrees with the prediction of GTR with an uncertainty of 0.2%. In 1993, R. Hulse and J. Taylor received the Nobel Prize in physics for the discovery and analysis of this double pulsar.



Recent findings on this topic are presented in [41]. The system PSR J0737-3039 A/B, described there, consists of two neutron stars, both of which are pulsars, and thus enable checking the predictions of GTR to an unprecedented accuracy.

## 110 Problems and Suggestions

1. Check the numbers in the last column of the table of section **I1** using the values from Einstein's formula on the same page.
2. Check the numerical values claimed in the last few lines of text on the Hipparcos satellite at the end of section **I2** !
3. In the Schwarzschild metric the diameter of the circle multiplied by Pi, is greater than the corresponding circumference. Epstein's bump yields an additional precession of the perihelion in the direction in which the planet orbits (**I1**). How could you realize with paper a geometry in which the circumference is greater than the diameter times Pi? How would that impact the additional precession of the perihelion?
4. Show that the value of  $\alpha = G \cdot M/c^2$  for the sun in units of 'light seconds' is about  $4.9261 \cdot 10^{-6}$ . What is the value of the gravitational constant G in these units?
5. We draw the cross section of the rolled up space-time as described in section **H5**. On the inside of the mass we have a spherical sector - and on the outside? For weak fields we can assume as a good approximation that  $y$  decreases there as a function of type  $y = a + b/x$ . Determine that solution  $f(x)$ , where the  $x$ -position of the inflection point is  $(3/4)$ ! What is the physical meaning of the  $x$ -Coordinate 3, and what is the value of  $R_s$ ? Is it still a weak field? Compare clocks on the surface of the body with those in the OFF!
6. When photons rise in a gravitational field they lose energy. Will they be slower or faster for an observer in OFF?
7. How many extra seconds elapse in OFF when in Amsterdam precisely a century has elapsed? Consider the GTR (gravitation of the sun and the earth) and the STR (orbital speed of the earth).
8. Determine based on the middle diagram of section **I6** the average speed of the airplane during the 15-hour flight!
9. (Section **I4**) Determine the relative velocity  $v$  which belongs to a Doppler frequency shift of  $\Delta f/f$  of  $2.22 \cdot 10^{-15}$ .
10. The nearly circular orbits of the NAVSTAR GPS satellites are so chosen that each satellite orbits the earth exactly twice during a sidereal day.
  - a) At what altitude above sea level do these satellites orbit?
  - b) By how many nanoseconds per orbit do the satellite clocks run fast compared to a clock at sea level at the equator, when the orbit passes over the poles?
  - c) Same as b) but for an orbit in the equatorial plane with the same rotational direction as the earth?
  - d) Same as c) but in the opposite rotation direction?
11. Visit the web-site <http://homepage.univie.ac.at/Franz.Embacher/Rel/> . You can find there programs for STR and GTR. Work through some of them - you should now be well prepared to do so!
12. Search for "Michael Kramer" and "PSR J0737-3039 A/B" and read the web infos about this binary pulsar system that has turned out to be a perfect 'laboratory' to test GTR in the case of strong gravitational fields.



An Einstein poster in the background viewed through one of the high-precision quartz spheres used in the Gravity Probe B experiment as a gyroscope. It is depicted similarly to a convex lens.

<http://einstein.stanford.edu/> or <http://einstein.stanford.edu/gallery/>



IR-View of "Pillars of Creation" at Centre of Eagle Nebula  
(VLT ANTU + ISAAC)

ESO PR Photo 37b/01 (20 December 2001)

© European Southern Observatory

## K Some Additional Topics

In this chapter we give brief references to supplementary topics and present ideas for possible further study. The actual work will be left to the reader. The topics are diverse such as historical attempts to measure the speed of light, considerations concerning 'natural' units for physical entities, theoretical considerations for calculations of future travel to neighboring fixed stars, the use of alternative mathematical formalisms for the STR, representing STR with other diagram types and the difference between measuring and seeing in the STR. Many topics have ended up here – many which I would have gladly dealt with directly but which would have given the book a discouraging length. Furthermore, you should now be very well prepared to tackle these topics independently!

## K1 Early Experiments to Measure the Speed of Light

The speculative-philosophical phase of discussion on the speed of light lasted until 1676. Only Galileo attempted, shortly after 1600, an experimental clarification of the issue, but with his lanterns and helpers stationed on opposing hills, he had no real chance of getting a result. He also correctly interpreted that the speed of light is much larger than the speed of sound. René Descartes on the other hand, put all of his prestige on the line with a (weak) argument for an infinitely large speed of light.

Presentations on the topic 'speed of light' might comprise one of the following areas:

- The speculative phase before 1676: Empedocles, Aristotle, Heron of Alexandria, the ancient Indians, Avicenna and Alhazen, Kepler, Francis Bacon and Descartes
- The experiments to measure the speed of light by Galileo and his pupils
- Ole Römer's explanation of the annual 'late' and 'early' arrival of the eclipse of Jupiter's moon Io (1676). His declaration to the speed of light: 22 minutes for the diameter of the earth's orbit. For the latter there were only rough estimates.
- James Bradley and the aberration of light (1728). His acknowledgment of the value of Römer's based on a very different measurement provided the breakthrough for the finiteness of the speed of light. Bradley's value for  $c$  was very close to the present value.
- Armand H.L. Fizeau was the first to measure in 1849 the speed of light for a distance of a few kilometers (Gear wheel method)
- Léon Foucault needed only a light path of a few meters in 1850 for his revolving mirror method and thus was able to measure the speed of light in different media
- The importance of the almost blind physicist François Arago as a supplier of ideas for Fizeau and Foucault
- P. Newcomb and A.A. Michelson improved Foucault's method in 1926 and measured the speed of light to about 0.002% accuracy.
- Definition of the speed of light (and thus in particular the length of a meter) in 1983 by the members of the International Bureau of Weights and Measures (BIMP) to 299,797,458 m/s

Also interesting is the experiment of Fizeau concerning the speed of light in flowing water (1851). He could not understand his results because he assumed the classical addition of velocities. Applying the addition of velocities according to STR yields his results immediately (see e.g., [25-103ff], [19-80f] or [14-120]).

The *definition* of the speed of light from 1983 opens the way to the ideas outlined in next section: For  $c = 1$  (by definition) we obtain a new system of units, which takes into account the very core of the STR!

## K2 Natural Units of Measurement in the STR and GTR

If one consistently thinks in space-time units, then it is clear that one must assign the speed of light the unit value 1: One space-time unit per space-time unit. One light-second per second, when expressed in time units. Thus energy, mass and momentum automatically receive the same units,  $E = m \cdot c^2$ , for example, yields simply  $E = m$ . This approach is the basis for all of the following proposals. Study the simplifications that result for the relationships we presented in **E5**!

Carl Friederich Gauss had already proposed eliminating the fundamental electromagnetic constants  $\epsilon_0$  and  $\mu_0$  by choosing the unit system such that they receive the value 1. This greatly simplifies the theory of electromagnetism. Given Maxwell's equation  $c^2 = 1 / (\epsilon_0 \cdot \mu_0)$  it thus also follows that  $c = 1$ , which Gauss could not have known. According to this proposal by Gauss, the electric and magnetic fields are measured in the same units. And it makes sense: According to STR, these two fields can be directly transformed into one another!

One can go even further: Setting the gravitational constant  $G$  to the unit-free value of 1, eliminates the kilogram and allows masses to be measured in time or length units. Especially nice is the use of length units, a mass is then just as heavy as its Schwarzschild radius  $R_S$ !

Setting even the Boltzmann constant  $k$  to the value 1 yields a new temperature scale and allows all physical quantities to be expressed, for example, in cm! This unit system is consistently used in [29].

One need not necessarily take such a radical approach. In any case it makes sense to apply  $\epsilon_0 = \mu_0 = c = 1$ . Study for each of the following, how the units of time, length, mass, acceleration, force, momentum and energy convert into those of our conventional MKS system. Determine in each case the corresponding values of the 'fundamental constants'. This should give you a whole new feeling for the dependence of these values on each other!

1. Times and lengths in seconds, masses in kilograms. What value does the gravitational constant  $G$  have?
2. Times and lengths in nanoseconds and the value of the gravitational constant 1. What would the mass of one kilogram be?
3. Times and lengths in centimeters and the value of the gravitational constant 1. What is a kilogram equivalent to? How heavy is the sun?
4. Same as 3 but also the Boltzmann constant is assigned the value 1. What is one degree Kelvin equivalent to?

Perhaps you may add an appropriate definition of electric charge so that the electric and magnetic units are also included!

"What makes Einstein's theory of relativity remarkable is its ability to unify various ideas in physics that had previously been treated independently. It unifies electricity with magnetism, materials [perhaps better: mass] with energy, gravity with acceleration and space with time." [45-25]

### **K3 General Formulas for Velocity Addition, the Doppler Effect and Aberration**

In sections **D4** and **D6** we considered how to add velocities in x direction in the STR. We also determined the correct formula for the Doppler Effect when the source moves in a direct line either toward or away from the observer. In Section **D5**, we saw how 'transverse' velocities transform and we also derived the angle of aberration one must take into account when the observer moves perpendicular to the segment connecting the source and observer.

These results are (important) special cases of more general results, which we could easily derive given our previous work. We will forego doing so and simply present references for further reading. Perhaps you may want to find your own derivation?

#### **General formulas for the addition of velocities:**

- by Einstein himself: [09-140ff]
- by Horst Melcher: [27-37ff]
- by Michael Fowler: [24-69f]
- by Roman Sexl and Herbert Schmidt: [25-100ff]
- by Jürgen Freund: [26-73ff]

#### **General formulas for the Doppler Effect:**

- by Einstein, without calculation: [09-146ff]
- by Horst Melcher: [27-72ff]
- by Jürgen Freund: [26-117ff]
- by Edwin Taylor and John Archibald Wheeler with four-vectors: [11-263]

#### **General formulas for Aberration:**

- by Einstein, without calculation: [09-146ff]
- by Horst Melcher: [27-74ff]
- by Jürgen Freund: [26-89ff]

## K4 Force and Acceleration in the STR

How are accelerations and forces transformed from one inertial frame to another? For example, the derivation of the transformations of the electric and magnetic field depends heavily on one already knowing how the forces transform; the Lorentz force law  $\mathbf{F} = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$  should turn into  $\mathbf{F}' = q \cdot (\mathbf{E}' + \mathbf{v}' \times \mathbf{B}')$ .

From the definitions  $\mathbf{a} = d\mathbf{v} / dt$  and  $\mathbf{a}' = d\mathbf{v}' / dt'$ , and the transformations of velocity and time we could deduce how accelerations transform in the STR. Since (by definition) the relation  $\mathbf{F} = d\mathbf{p} / dt$  continues to apply and since we already know how to transform masses and velocities, we could also infer how momentum is transformed. Taking the derivative with respect to time we would obtain the transformation formulas for force.

A more elegant solution is provided by four-vectors (see also **K9**). This tool provides a higher yield with a smaller algebraic effort! However, one must beforehand be comfortable using these vectors. We give references for both paths:

### Transformation of forces and accelerations without four-vectors:

- by Horst Melcher: [27-45f]
- by Jürgen Freund: [26-95ff]
- ??

### Transformation of forces and accelerations with four-vectors:

- by Roman Sexl and Herbert Schmidt: [25-115ff]
- by Jürgen Freund: [26, chapters 28 to 33]
- ??

## K5 The "Conquest of Space"

It is a nice exercise in mathematics and physics to calculate a human voyage to a nearby star. Human, here, means that during the acceleration at the start and during the deceleration before the return the passengers should feel a constant acceleration equivalent to the gravitational force felt on the surface of the earth. Given the desired cruise velocity after the acceleration phase and the distance to the destination then one has all of the information needed.

Kranzer investigates in [43] such a trip to the nearest star  $\alpha$  Centauri (distance about 4.2 light years) at a speed of  $0.9 \cdot c$  after the acceleration phase. He uses a few formulas without showing their derivation. For interested students this is a nice challenge! Calculus at the high school level is sufficient to do the calculations. The starting point is the following equation (see also section E4!)

$$m_0 \cdot g = \text{konstant} = \frac{dp}{dt} = \frac{dp}{dv} \cdot \frac{dv}{dt} = m_0 \cdot \frac{d}{dv} \left( \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \right) \cdot \frac{dv}{dt} = m_0 \cdot \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{3}{2}} \cdot \frac{dv}{dt}$$

After division by  $m_0$  one obtains for the speed of the space vehicle, the following differential equation

$$g \cdot \left( 1 - \frac{v^2}{c^2} \right)^{\frac{3}{2}} = \frac{dv}{dt}$$

Students can at least verify that the following functions for  $v(t)$  and  $x(t)$  satisfy this differential equation:

$$v(t) = v_0 + \frac{g \cdot c \cdot t}{\sqrt{c^2 + g^2 \cdot t^2}} ; \quad x(t) = x_0 + v_0 \cdot t + \frac{c^2}{g} \cdot \left( \sqrt{1 + \frac{g^2 \cdot t^2}{c^2}} - 1 \right)$$

The space traveler's elapsed proper time during the acceleration phase is then calculated by taking into account that the following always applies

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}} \cdot dt$$

Forming the integral on the right side for the duration of the acceleration phase in coordinate time  $t$  for an observer on the earth, yields the elapsed proper time  $\Delta\tau$  for a passenger in the spaceship. The result is (for  $v_0 = 0$  and  $x_0 = 0$ )

$$\Delta\tau = \frac{c}{g} \cdot \left[ \ln \left( t + \sqrt{\frac{c^2}{g^2} + t^2} \right) \right]_{t_0}^{t_1}$$

The journey there and back takes around 12 years for the earth-bound people, while somewhat more than three and a half years elapse on-board. How to achieve technically a cruise velocity of  $0.9 \cdot c$  for three-quarters the start mass (!) remains an unsolved miracle. To speak of an impending "conquest of space" is extremely exaggerated.

For people who like to do calculations, [27] contains a lot of stimulating material on pages 164 to 230! Also [25-161ff] discusses what the STR has to say about travel in our cosmic neighborhood.

## K6 Alternative Derivation of $E = m \cdot c^2$

We derived this formula in **E4** in the usual way using kinetic energy and the amount of work invested in the acceleration of the mass. Similar to the Pythagorean Theorem, however, there are many proofs and derivations of this famous equation. I have already alluded to probably the most beautiful in the last paragraph of section **E5**. In this case Einstein used the conservation of momentum and the conservation of energy as well as the formulas for the energy and the momentum of photons. It is presented in [22-98ff].

Some other nice proofs (see [26-55ff], [15-131ff]) also use the momentum of light particles or the radiation pressure exerted by electromagnetic wave activity. This quantity was known in 1880 (i.e., 'long' before quantum theory) from the theory of Maxwell.

Max Born chose a different approach to represent Einstein's relativity theories in his book [42] for the general public first published in 1920. He derives the formula from the inelastic collision of two clumps moving at non-relativistic velocities, i.e., exactly as shown in **E3**. This method is, in principle, the same used by Sexl et al. in [11-24]: The formula for the increase in mass after its having been accelerated is expanded as a power series, whose fourth and higher order terms of  $v/c$  are dropped, thereby yielding  $E_{\text{kin}} = \Delta m \cdot c^2$ .

Einstein used a similar approach in September 1905. In his essay entitled "Does the Inertia of a Body Depend on Its Energy Content?" he derives the famous equation from his transformation formulas for radiation energy, which we have not discussed. The essay is only four pages, but it can not be recommended for general reading. However, we can understand the last sections, in which Einstein shows that he is quite aware of the general implications of his formula. In the quote below, we have replaced the character  $L$ , which Einstein still used at that time for energy, with  $E$  and for the speed of light we have written  $c$  instead of  $V$ :

"If a body emits the energy  $E$  in the form of radiation, its mass decreases by  $E/c^2$ . Here it is obviously inessential that the energy taken from the body turns into radiant energy, so we are led to the more general conclusion:

The mass of a body is a measure of its energy content; if the energy changes by  $E$ , the mass changes in the same sense by  $E/(9 \cdot 10^{20})$  if the energy is measured in ergs and the mass in grams.

It is not excluded that it will prove possible to test this theory using bodies whose energy content is variable to a high degree (e.g., radium salts).

If the theory agrees with the facts, then radiation carries inertia between emitting and absorbing bodies."

Bern, September 1905. [09-164]

Original publication in "Annalen der Physik", vol. 18 [1905], p.639-641. All of the works of Einstein in the journal "Annalen der Physik" can be downloaded as pdf photocopies at <http://www.physik.uni-augsburg.de/annalen/history/Einstein-in-AdP.htm>



We do not need all of the following segments. Of course, their physical meaning is also irrelevant for the proof. However:

AK = AG = BL = CM = EO	time elapsed for Red
AB = FG = KL = CD = QO	length of Blue's path for Rd, proper length of this path
AF = BG = CH = EI	time elapsed for Blue
CE = HI = MO	length of Blue's path for black
CI = CQ = CP	time elapsed for Black

The calculation is then quite simple:

$$\begin{aligned} \sin(\gamma) &= \frac{CE}{CI} = \frac{MO}{CQ} = \frac{MN+NO}{CN+NQ} = \frac{CM \cdot \tan(\alpha) + QO / \cos(\alpha)}{CM / \cos(\alpha) + QO \cdot \tan(\alpha)} = \\ &= \frac{AG \cdot \tan(\alpha) + AB / \cos(\alpha)}{AG / \cos(\alpha) + AB \cdot \tan(\alpha)} = \frac{AG \cdot \tan(\alpha) + AG \cdot \sin(\beta) / \cos(\alpha)}{AG / \cos(\alpha) + AG \cdot \sin(\beta) \cdot \tan(\alpha)} = \\ &= \frac{\tan(\alpha) + \sin(\beta) / \cos(\alpha)}{1 / \cos(\alpha) + \sin(\beta) \cdot \tan(\alpha)} = \frac{\sin(\alpha) + \sin(\beta)}{1 + \sin(\beta) \cdot \sin(\alpha)} = \frac{\sin(\alpha) + \sin(\beta)}{1 + \sin(\alpha) \cdot \sin(\beta)} \end{aligned}$$

The interpretation of this calculation is left to the reader: If one assumes that we have already proven the formula for addition of speeds, then these calculations should check the correctness of the above construction of the angle  $\gamma$ . If one accepts the correctness of the above drawing, then the calculation gives a proof for the addition of speeds, independent of the Lorentz transformations!

It requires some experience dealing with Epstein diagrams to recognize the correctness of the construction free of doubt. So it is perhaps not the didactic approach of first choice. Nevertheless, the drawing of Epstein shows clearly some of the most important aspects: Since the proper time of the sequence for Blue is an invariant ( $AF = BG = CH = EI$ ), then the addition of two velocities each smaller than  $c$ , will always yield a speed which is itself also smaller than  $c$ . If Blue moves horizontally with respect to Red (i.e., such as light with velocity  $c$ ), then Blue will also move horizontally with respect to Black:  $v \oplus c = c$ .

I would like to reiterate my thanks to Alfred Hepp for drawing my attention in the first place to appendix A in the second edition of [15]. He also persistently encouraged me to expand the construction of Epstein into a proof of the addition of speeds formula based solely on Epstein diagrams. Dissatisfied with my first proof, he also made significant contributions resulting in the present, much simpler derivation.

Another suggestion: The  $\oplus$ -addition makes the open interval  $(-1, 1)$  a commutative group. The identity element is zero and the inverse of  $v$  is  $-v$ . The addition is also monotonic in the sense that if  $a < b$  then it follows that  $a \oplus d < b \oplus d$ . Prove twice that  $\oplus$ -addition is associative: a) by formal algebraic calculation and b) by physical interpretation. Why must the two endpoints 1 and -1 be excluded?

## K8 The Transformation of Electromagnetic Quantities

We should again emphasize that our representation of the STR is incomplete: The implications for electricity and magnetism are not discussed, although the impetus for the development of this aspect of the STR came from this corner. Once again I refer (see F6) to the work on this subject by other authors.

### **STR and Electromagnetism in elementary presentation**

- by Michael Fowler [24]
- ??
- ??

### **STR and Electromagnetism with four-vectors**

- by Roman SEXT and Herbert Schmidt [25-177ff]
- by Jürgen Freund [26, chapter 34]
- ??

### **STR and Electromagnetism with vector analysis**

- by Albert Einstein [09-142ff]
- ??
- in any university level text book

## K9 SRT with Four-Vectors

Geometric problems can be treated with many different mathematical techniques. For example, you can calculate the volume of a tetrahedron using basic geometry, using vector geometry or using integral calculus. Certain questions can often be elegantly answered through the appropriate approach, while another approach would be complicated or provide only approximate success.

The best methodology for doing algebraic calculations in the STR is the one with four-vectors! Not only the place and time of an event, but also all other physical quantities are consistently described by vectors with 4 components: There are the four-speed, four-acceleration, the four-force, the four-momentum and the four-current vectors. All of these four-vectors transform themselves according to the Lorentz transformations in the transition to another coordinate system, just as we have seen with location and time coordinates. And for any four-vectors **A** and **B**, there is a simple scalar product **A** · **B** which yields a constant value, independent of the reference system!

Let us take as examples the four-momentum **P** and the four-speed **V**:

$$\mathbf{P} = (E_{\text{tot}}/c, p_x, p_y, p_z) = m_0 \gamma \cdot (c, v_x, v_y, v_z) = m_0 \cdot \mathbf{V} \quad \text{or as shorthand}$$

$$\mathbf{P} = (E_{\text{tot}}/c, \mathbf{p}) = m_0 \gamma \cdot (c, \mathbf{v}) = m_0 \cdot \mathbf{V}$$

Where **p** is the 3d-momentum vector and **v** the 3d-velocity vector. The scalar product of two four-vectors  $(x_0, x_1, x_2, x_3)$  and  $(y_0, y_1, y_2, y_3)$  is defined by  $x_0 \cdot y_0 - x_1 \cdot y_1 - x_2 \cdot y_2 - x_3 \cdot y_3$ . Quickly computing **P**<sup>2</sup> and **V**<sup>2</sup> using this definition of the square of a vector shows:

$$\mathbf{P}^2 = (E_{\text{tot}}/c)^2 - \mathbf{p}^2 = (E_{\text{tot}}^2 - \mathbf{p}^2 \cdot c^2)/c^2 = E_0^2 / c^2 = m_0^2 \cdot c^2 \quad \text{following our equations in E5 !}$$

This is obviously an invariant quantity. Considering the Epstein diagram in **E5**, if you divide all sides of the right triangle by *c*, you see that this calculation is just a variant of the Pythagorean Theorem. I would argue that Epstein diagrams and four-vector arithmetic are closely related!

We determine **V**<sup>2</sup> for the case where **v** = *v<sub>x</sub>* and thus *v<sub>y</sub>* = 0 = *v<sub>z</sub>* :

$$\mathbf{V}^2 = (1/\gamma)^2 \cdot (c^2 - v_x^2 - 0 - 0) = (c^2 - v^2)/(1 - v^2/c^2) = c^2 \cdot (1 - v^2/c^2)/(1 - v^2/c^2) = c^2$$

Again, this is obviously an invariant, i.e., a value independent of the reference system. This result agrees beautifully with Epstein's 'Myth'! As a small exercise you might consider what **P**·**V** means.

The aim of this book was not the algebraic treatment of challenging (and important) examples such as the Compton scattering. Its primary goal was to provide a view on the statements made by the STR and GTR. Or, as Epstein writes: "To understand the Special Theory of Relativity at the gut level, a good myth must be invented" [15-78]. To communicate Epstein's myth was my main concern. Once this basis has been attained, it is easy, in a second pass, to acquire some of the technical tools. These tools can then be used to address arbitrarily tricky problems. And the tool for the calculations in special relativity is the algebra of four-vectors.

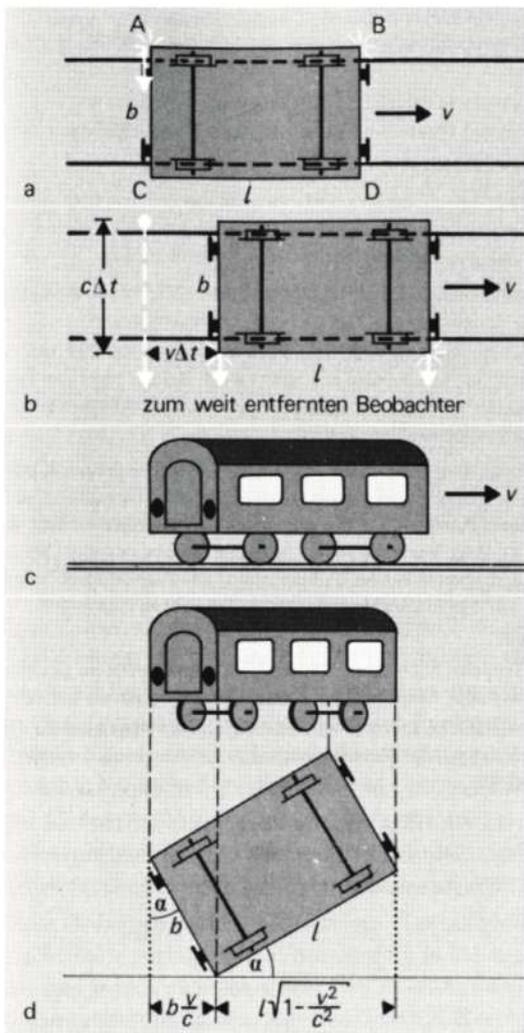
For an initial study of four-vectors [25] is recommended. If you work through chapters 12 and 13, you will already have made a significant start. Also, the presentation in [26] is perfectly accessible for someone with a solid high school background. Its title "Special Relativity for Beginners: A Textbook for Undergraduates" describes the level well. The sections on four-vectors in [14] and [19] are restricted to the energy-momentum vector and do not introduce the full power of this concept.

Or take a look at <http://www.relativity.li/en/maxwell2/str-maxwell-equations/>

## K10 Measuring and Seeing in the STR

Measuring and seeing are not the same thing. To take a measurement typically means to 'stop' time using a clock *at the location of an event*, that is, to capture a moment at a given location. Vision is a process by which at a given moment and a specific location we register all of the optical signals, which arrive from various places and which originated at various times. Seeing is therefore comparable to photography. In an astronomical photograph of the new moon with an exposure time of 0.1 seconds, we see how it looked about a second ago; the planet Mars, as it was a quarter of an hour ago; and for Saturn we see the light which was reflected about an hour ago! If we increase the exposure (a few minutes are sufficient), then perhaps we can even detect a galaxy, revealed by photons which started their journey millions of years ago!

Thus it is possible to see things that no longer exist. A supernova in the Large Magellanic Cloud, a small companion galaxy of our Milky Way, occurred about 163,000 years ago, when we observe it today. The travel-time of the light must also be considered when thinking about how an object appears which is moving very quickly. Consider the following diagram from [25-85]:



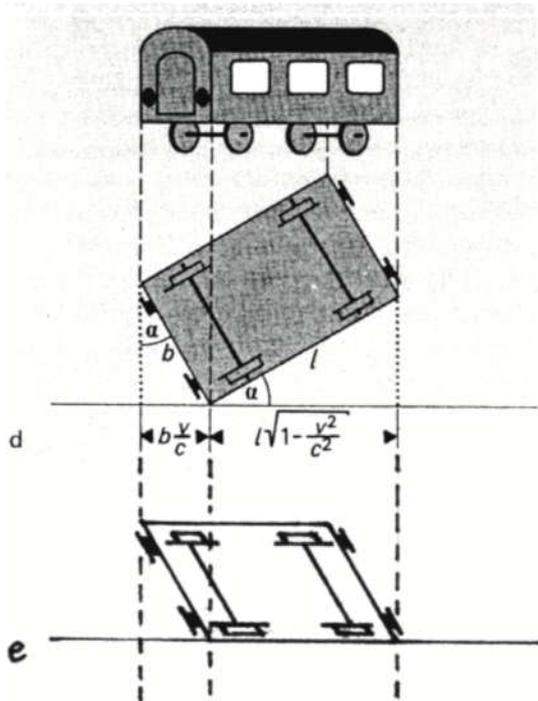
Photons leave the corners A and B of the rail car. However, the photons from B cannot reach us.

Only now (after time  $\Delta t$ ) are the photons from the front corners C and D sent, which reach our eye simultaneously with those from A.

The car would appear thus if Lorentz contraction did not exist!

We actually see the car thus, since the distance between the corners C and D has shrunk due to the high velocity  $v$ .

That is precisely the view of the car we would have, if he had turned away from our line of sight at the angle  $\alpha$ , where  $\sin(\alpha) = v / c$ .



The fact that the car appears to have turned away is only due to our 3D interpretation of the 2D image we have of the situation!

It would be equally correct to say that the car has been tilted and Lorentz contracted. However, we are not used to this interpretation of a visual impression. But it would fit much better to the transformation of the density!

Of course, this comment is not from the author of [25].

Meanwhile, there is a group within the physics community, which is using the computing power available today to show how, for example, a flight through the Brandenburg Gate at  $0.95 \cdot c$  would be visually experienced. Sometimes even the change in color due to the optical Doppler effect is taken into account. Clearly, one must play the whole thing back in slow motion, so that it does not run too fast for a human observer. A good location for such visualizations is

[www.tempolimit-lichtgeschwindigkeit.de](http://www.tempolimit-lichtgeschwindigkeit.de)

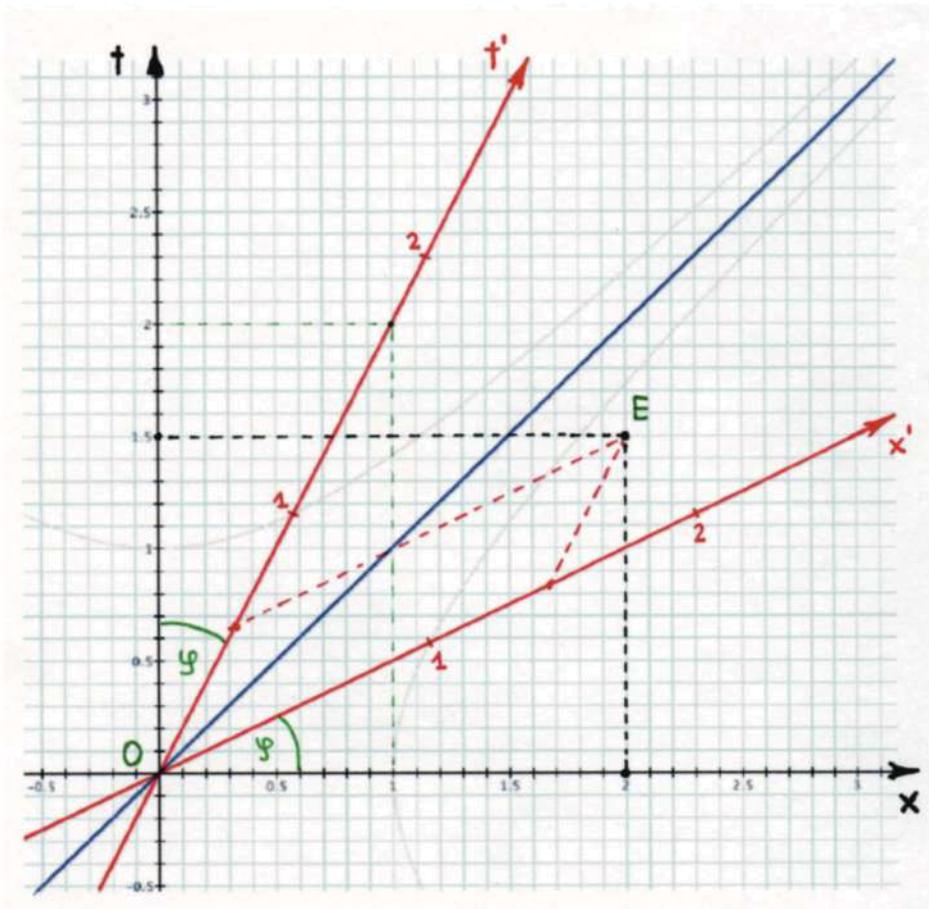
Ute Kraus of the University of Tübingen is the person behind this address. Her group realized for the Einstein Jubilee of 2005 relativistic bicycle tours through the old towns of Bern and Tübingen. The exhibitions in Ulm and in Bern even offered the visitor to mount a real (stationary) bicycle and pedal away. The pedal speed was then processed by a computer which amplified the effects of 30 km/h to 300,000 km/s. To make it possible to navigate the narrow streets (and still be able to see something!) the houses passed by at the unamplified velocity.

If you delight in such visualizations, then visit the website recommended above. You will also find references and other related material about the theories of relativity of Albert Einstein.

## K11 SRT and Minkowski Diagrams

Epstein diagrams are not used in any of the STR introductions known to me - except of course in the original [15] by Epstein himself. Most books use Minkowski diagrams when graphically displaying the situation of two inertial frames moving relative to each other. These Minkowski diagrams obviously have certain advantages, but I think Epstein diagrams are more suitable for a first and even a second exposure to the STR. Epstein diagrams are entirely indispensable if additionally one wishes to get a concrete feel for the GTR. That is the main reason I have written this book.

I will highlight here only the significant differences between the Minkowski diagrams and Epstein diagrams and leave the details to the numerous reference books which include an introduction to the use of Minkowski diagrams.

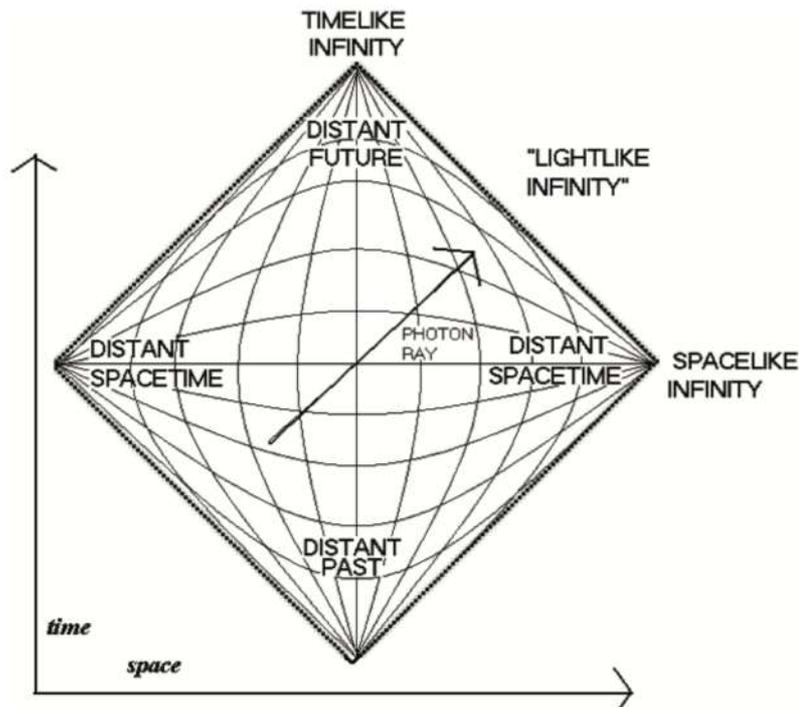


Here, Red moves with  $v = 0.5 \cdot c$  along Black's  $x$ -axis (note the helping green dashed line). The Minkowski diagram generally uses  $\tan(\phi) = v/c$ , where we have  $\sin(\phi) = v/c$  in the Epstein diagram. The event E takes place at point 2 and at time 1.5 for black. Red assigns event E approximately the coordinates  $x' \approx 1.45$  and  $t' \approx 0.58$  (the dashed red lines are parallel to the red coordinate axes). The tick marks are shown on the light gray calibration hyperbolas  $y^2 = 1 + x^2$  and  $x^2 = 1 + y^2$  respectively. Light particles move in the inertial frame either perpendicular or parallel to the blue angle bisector. In Epstein diagrams, on the other hand, light particles move in each frame perpendicular to its time-axis.

Almost all introductions to the STR which are not limited to a qualitative description of the phenomena also present Minkowski diagrams. In our bibliography examples are [14], [19], [25] and [26]. But the most beautiful introduction to Minkowski-Diagrams is given by Sander Bais in [44] !

## K12 SRT and Penrose Diagrams

There are many other possibilities to graphically represent relativistic relationships. The two British mathematicians B. Carter and R. Penrose have a model in which the infinitely extended Minkowski plane is projected onto a square in which light beams continue to be straight lines running parallel to one of the two angle bisectors. Lines of constant time or constant location become hyperbolas. In the center of the picture one sees the undistorted current local event, while distant events are compressed together:



This figure is taken from the free encyclopedia Wikipedia. On the website of Franz Embacher

<http://homepage.univie.ac.at/Franz.Embacher/Rel/>

you can find a Java applet, which allows you to play with this coordinate transformation. If you play with it seriously and use it to solve the problems that are suggested, then you gain a good understanding of what this model offers.

The arc tangent is used since it is a monotonically increasing function; approximates the identity function in the vicinity of the origin (i.e.,  $f(x) \approx x$ ); and has a finite limit as  $x$  approaches  $\infty$ . Thus the corners of the square lie  $\pm \pi/2$  from the origin. The mapping of the Minkowski plane into the Penrose-world is defined by the following equations:

$$x' + t' = \text{ArcTan}(x + t) \quad \text{and} \quad x' - t' = \text{ArcTan}(x - t)$$

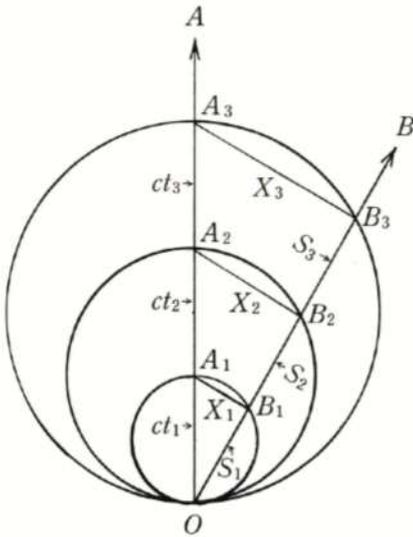
Addition (and subtraction) of these equations immediately gives the transformation equations for  $x'$  and  $t'$  respectively. It is easy to show that light beams that go out from the origin in the Penrose diagram are angle bisectors. Show that all the light beams run parallel to these!

Also: Find a model yourself, which has similar properties! Is the arc tangent really better than your model?

## K13 SRT and Asano Diagrams

After 'completing' this book I discovered in the basement of the Central Library in Zurich a little book [45], in which the two brothers Seiichi and Shiro Asano present their "Space-Time Circular Diagrams". The first Japanese edition was published in 1983, thus coinciding with the first edition [15] by Epstein! The brothers Asano, as small boys, were impressed by all of the whoopla of Einstein's visit to Japan in 1922. They went on to have careers as an electrical engineer and a physician, respectively. After retiring, they decided to elucidate the STR for themselves and others.

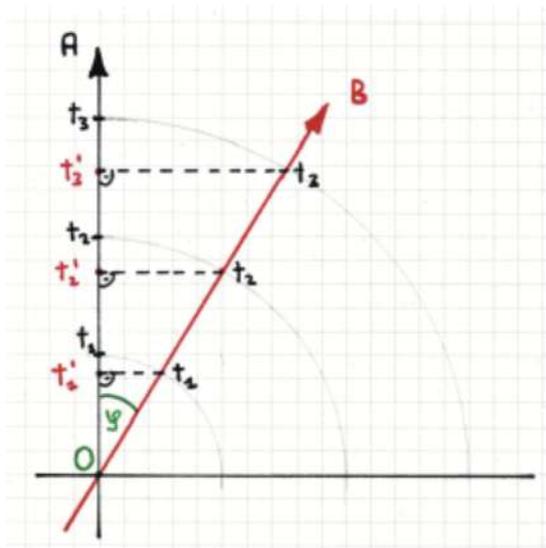
Like Epstein they take as their starting point Minkowski's equation  $\Delta\tau^2 = c \cdot \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$  (times and lengths are measured in the same units), suppress the y and z component and rearrange the remainder of the relationship  $\Delta\tau^2 = c \cdot \Delta t^2 - \Delta x^2$  so that it can be interpreted as the equation of a circle:  $c \cdot \Delta t^2 = \Delta\tau^2 + \Delta x^2$ . Also with the Asano brothers the straight line on which B moves with constant velocity v through space-time is tilted with respect to the time-axis of a stationary observer A by an angle  $\phi$ , where  $\sin(\phi) = v / c$ . On [45-49] they consider right triangles that are congruent to those in the corresponding diagrams of Epstein.



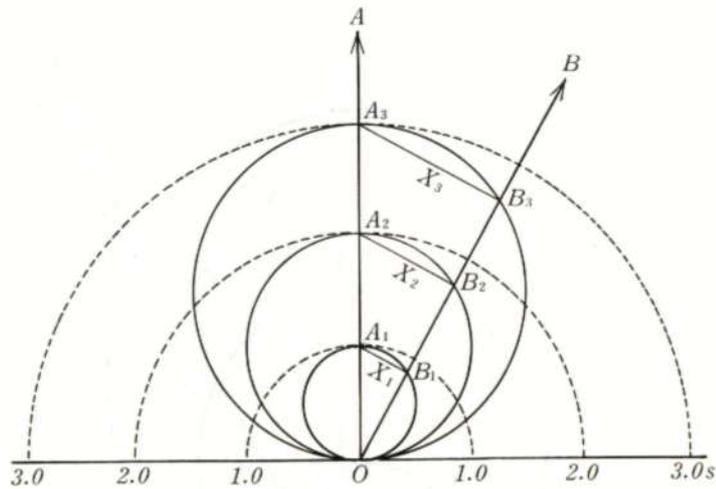
A and B have met at O and both have set their clocks to zero. The sine of the angle AOB is  $v/c$ . At  $B_1, B_2$  and  $B_3$ , we have right angles according to the theorem of Thales.

When the time  $t_3$  has elapsed for A, which corresponds to the distance  $OA_3$ , then for B the time which corresponds to the distance  $OB_3$  has elapsed. At time  $t_3$ , B is the distance  $X_3 = A_3B_3$  from A.

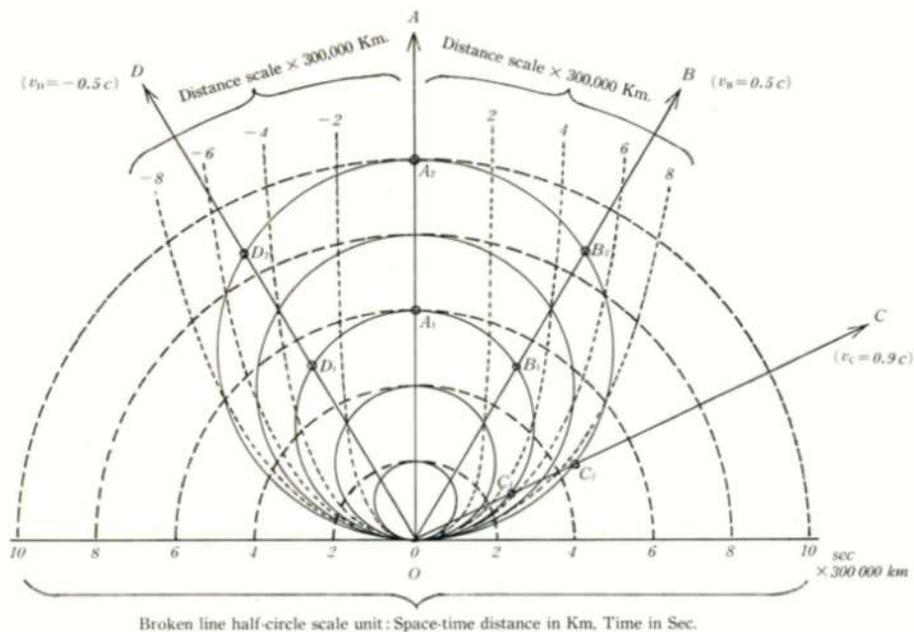
We obtain the corresponding congruent right triangles of the Epstein diagram by reflecting those of the Asano diagram through the angle bisector of AOB.



Also the Epstein diagrams characteristic semicircles around O occasionally appear in the Asano diagrams; however, they indicate only the elapsed time intervals for A. The diagrams show a dilation with center O and a stretching factor, which is proportional to time [45-50]:



But for the spatial axes the brothers have no good solution. One might say that they still tried to separate time, space-time and space. The dashed curves, which indicate at what speed a certain distance (in light-seconds) is reached, are quite complicated:



Do you notice the point which belongs to the Pythagorean triple (6/8/10) and which also lies on the straight line for  $v = 0.6 \cdot c$ ?

Epstein diagrams are clearly preferable to those of Asano. They are based on a simple postulate and they are more easily drawn and read. But it is interesting to note that similar approaches appeared in different places simultaneously. The Asano brothers do not mention their 'competitor' Epstein in the first English edition of 1994.



The Starburst Galaxy NGC 908  
(FORS/VLT)

ESO Press Photo 27a/06 (26 July 2006)

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## L Bibliography

Here you find the works referenced in square brackets. The first list **L1** is arranged according to the order of occurrence in the text. The second section **L2** orders the same titles alphabetically according to author, using the first mentioned author for titles with multiple authors.

The designation [15-158ff] refers naturally to the pages 158 and following in book [15].

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The accomplishments of **Johannes Kepler** for the development of modern astronomy and physics can not be overemphasized. On the principle of inertia, he was as close as Galileo; on the forces that must act between the heavenly bodies, he had clear ideas; and concerning kinematics he was far superior to Galileo. Also his observations on the tides are much more reasonable than those of Galileo. Kepler and Galileo corresponded. While Kepler spoke with great respect for Galileo's research, Galileo hardly even took note of Kepler's work and never provided him with one of his telescopes. Kepler had expressly asked him for one so that he could experience with his own eyes the wonderful discoveries made by Galileo. Also in optics theory, Kepler was far ahead of Galileo. Kepler's booklet "Dioptrics" is still today useful as a theory of ray optics.

Einstein writes of the relationship between Galileo and Kepler: "Alas, you find [vanity] in so many scientists! It has always pained me that Galileo did not acknowledge the work of Kepler." [17-79]

With the following excerpt from the introduction to "Astronomia Nova" of Kepler, I would like to convey an impression of his thinking in physics:

"The true doctrine of gravity is based on the following axioms: Every corporeal substance, insofar as it is physical, is naturally inclined to rest at the place where it finds itself, outside the force field of a related body. Gravity arises from the mutual corporeal tendency of related bodies for unification or combination (magnetic force also arises in this way), thus the earth attracts the stone rather than the stone seeks the earth. ...

If the earth were not round, then gravity would not attract in a straight line towards the center of the earth, but rather from different angles toward different points. If one places two stones any place in the world, close to each other but outside of the influence of a third body, then the stones would unite in an intermediate place like two magnetic bodies with one approaching the other by a distance, which is proportional to the mass of the other. ...

The range of attraction of the moon reaches to the earth and draws the water into the hot zone, to meet it where it reaches the zenith; imperceptibly in tightly enclosed seas, but noticeably, where the sea expanse is great and the waters have great leeway to flow back and forth. ..."

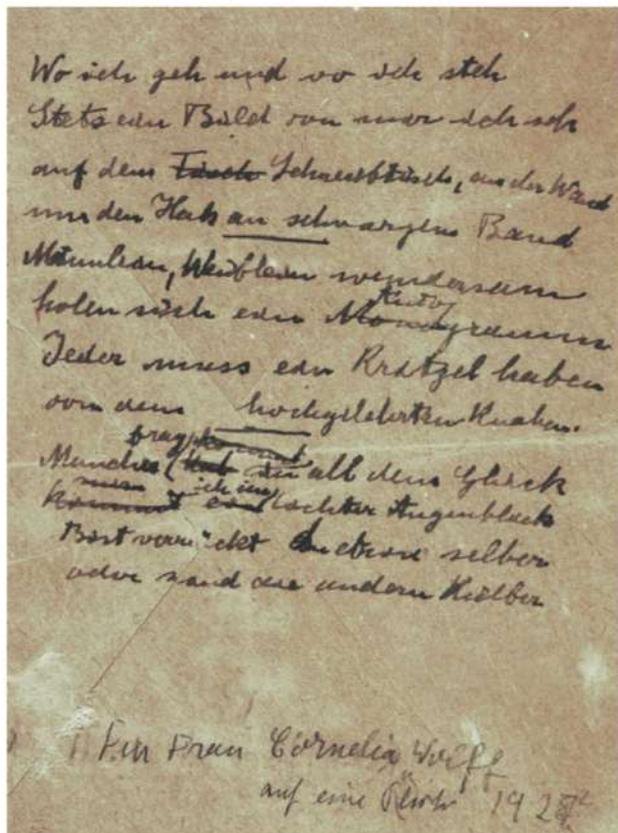
[translated by Samuel Edelstein from 08-25f]

What Kepler was lacking was a clear understanding of the dynamics of circular motion. He could not quite overcome the old idea that the circular orbit of heavenly bodies is natural and free of force. Christian Huygens was the first to capture the dynamics of circular motion and compute the correct value of the required centripetal force. Thus all the building blocks were available for Newton's great synthesis. "If I have seen further than others, it is because I have stood on the shoulders of giants" wrote Newton in February 1675 with well-calculated modesty in a letter to Robert Hooke (according to Wikipedia, the quote dates back to Didactus Stella). Kepler and Huygens were two of these giants. Galileo, Fermat, Descartes, Pascal, and Hooke were others who did much of the mathematical and physical preparation for Newton. What these few men were for Newton, the Greek geometers and astronomers were for Copernicus and Kepler.

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Wo ich geh und wo ich steh  
stets ein Bild von mir ich seh  
auf dem Schreibtisch, an der Wand  
um den Hals am schwarzen Band

Männlein, Weiblein wundersam  
holen sich ein Autogramm  
Jeder muss ein Kritzel haben  
von dem hochgelehrten Knaben.

Manchmal frag in all dem Glück  
ich im lichten Augenblick  
Bist verrückt du etwa selber  
oder sind die andern Kälber

für Frau Cornelia Wolff  
auf eine Photo 1927

Picture and text from [46-211]